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POLYNOMIAL SOLUTIONS OF PROBLEMS

IN BENDING OF FLAT PLATES

A THESIS

Presented to

the Faculty of the Graduate Division

by

Robert Benedict Sayer

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Aeronautical Engineering

Georgia Institute of Technology

June, 1959

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IN BENDING OF FLAT PLATES

Approved ______________________

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Date Approved by Chairman 1 June 1959

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SUMMARY

Problems of elasticity usually require solution of certain partial differential equations with given boundary conditions. Only in the case of simple boundaries can these equations be treated in a rigorous manner. Approximate methods must be used when rigorous solutions cannot be obtained, the most common method being the replacing of differential equations by finite difference equations. A method of approximate solution by use of polynomials is presented and compared with the finite difference method for the same net size. The approximate polynomial method is then applied to the problem of the wide cantilever plate with a concentrated load at the center of the free edge.

The finite difference method was used in an article by D. C. Holl (1) to obtain the deflections and internal bending moments in the wide cantilever plate problem mentioned above. Various inconsistencies in the equations written, and in the results obtained, have been noted. The main inconsistency is that the solution presented does not satisfy the system of equations shown, nor does it satisfy the system of equations obtained by eliminating the various apparent errors contained in the equations. This inconsistency is discussed in Chapter II, and a finite difference solution with Holl's apparent errors eliminated is obtained as solution one; the values of maximum deflection and maximum internal bending moment are somewhat less than the values shown in Holl's solution. Neither Holl's results nor those of solution one appear to be reasonably accurate by comparison with the exact solution for an infinitely wide cantilever plate as obtained by C. W. MacGregor (2). The major apparent inaccuracy is that Holl's results and those of solution one indicate

values of maximum internal bending moment which are two and five percent, respectively, less than MacGregor's value.

The appropriate polynomial method of solution of problems in elasticity is presented in Chapter III. A general equation of a polynomial with the values of the desired unknowns at various net points on the elastic plate is written; the derivatives of the polynomial as required by the problem, evaluated at the net points, are used to form a system of linear equations which is solved for the values of the unknowns. This method is used to obtain an approximate solution to several simple problems of elastic plate bending, the exact solutions of which are known; it is shown that the method yields more accurate results than the finite difference method, except for certain cases where the two methods are equivalent.

The problem of the wide cantilever plate with a concentrated load at the center of the free edge is solved by the approximate polynomial method in Chapter IV; the net size used is the same as that of Holl and solution one. A value of maximum internal bending moment which exceeds MacGregor's result by 33 per cent is obtained; this result seems to be more reasonable than that obtained by the finite difference method. Table 3 and Figures 4 and 5 present a comparison of the results obtained by the various methods at the net points.

It is concluded that the approximate polynomial method of solution gives more accurate results than the finite difference method for a given net size. It is recommended that the possibility of obtaining the approximate solutions to other problems involving ordinary or partial differential equations by use of this approximate polynomial method be investigated. High speed digital computers can be used to obtain the solutions to the large system of equations involved in a practical problem.

CHAPTER I

INTRODUCTION

The general theory of elasticity is concerned with stresses and strains in elastic bodies; literature covering the subject is extensive. It is stated by Timoshenko and Goodier (3) that "We have seen that the problems of elasticity usually require solution of certain partial differential equations with given boundary conditions. Only in the case of simple boundaries can these equations be treated in a rigorous manner." The text following states that approximate methods must be used when rigorous solutions cannot be obtained and mentions as one such method the replacing of differential equations by finite difference equations. This will henceforth be referred to as the "finite difference" method.

It is stated (4) that in order to increase the accuracy of the finite difference method, finer and finer nets must be used. This increases the number of equations which must be solved to the point where the solution of the system of equations eventually becomes impractical. Methods of successive approximation (4), including R. V. Southwell's relaxation method (5), can be used. However, development of digital computers and use of them to find the inverses of large matrices is presently such that the solutions to large systems of equations can be found with a relatively small amount of time and effort, as compared with that which would be required by a successive approximation method.

Examples shown in the literature (6) (7) are solved with reasonable accuracy by the finite difference method. A solution by this method of the problem of a wide cantilever plate with a concentrated load at the center of the free edge was presented in an article by D. C. Holl (1) in 1937. The article contains inconsistencies, the main one being that the results presented do not satisfy the system of equations as given, or as corrected for apparent errors. Solutions to a corrected system of equations, and to a slightly modified system, were obtained using a digital computer as solutions one and two. Neither solution seems to be consistent with what would be expected in several respects. The exact solution of this problem is not known.

An approximate solution by use of polynomials with undetermined coefficients is next presented; it is shown to be more accurate than the finite difference method for a given net size except for certain cases in which the two methods are equivalent. This polynomial method is then applied to the cantilever plate problem, and is shown to give results which appear to be more reasonable than those obtained by the finite difference method.

CHAPTER II

FINITE DIFFERENCE SOLUTIONS

Figure 1 is a reproduction of Figure 1 of the article by D. C. Holl (1). Holl's system of finite difference equations appear on pages 20 through 24 of the appendix. Several apparent errors have been noted, of which two appear to be typographical. The major apparent error is that the thirty-two of thirty-nine values of unknowns presented by Holl do not satisfy the system of thirty-nine equations. Twenty-six equations, those of groups [1], [3], [4], [6] and [7] do not involve the other seven unknowns, and are satisfied to a good degree of accuracy by the thirty-two values presented by Holl; however, values of the seven remaining unknowns obtained from seven of the remaining thirteen equations, when combined with Holl's thirty-two values, do not satisfy the remaining six equations. The values shown in Holl's article are not, then, thirty-two of the thirty-nine values of the unknowns which satisfy the system of equations.

The solution to the system of equations corrected for apparent errors was obtained using a digital computer. The value of Poisson's ratio was set equal to three tenths, as in Holl's solution, and the constants shown on the next page were substituted in the equations to permit solution by digital computer.

$$a = 2$$

$$\lambda = a/2 = 1$$

$$N = Eh^3/12(1-\nu^2) = 10^3$$

$$P = 250$$

This resulted in the system of equations shown on pages 28 and 29 of the appendix, which appear in the same order as Holl's equations. The solution to this system of equations, and the check on the accuracy of the solution, is shown on pages 30 and 31 of the appendix. Coefficients calculated from this solution and corresponding to those shown by Holl are presented as solution one in Table 3 on page 68. It may be seen that the values of the unknowns are significantly different, and that the deflections at points B and C are in fact of opposite sign to those shown by Holl. The following statements are made on page 8 of Holl's article, and make reference to an exact solution by C. W. MacGregor (2) for an infinitely wide plate:

> The maximum deflection under the load point is $w_d = 0.18733$ Pa^2/N which exceeds MacGregor's results by 12 per cent. The result of the special study in the Appendix of $w_d = .17902$ Pa^2/N is only 7 per cent greater than MacGregor's results. . . . . The maximum moment at the clamped edge nearest the load point shows a difference of only 2 per cent from MacGregor's results. However, the max. moment in the finite length of cantilever plate is actually smaller than in the infinite length, which perhaps is due to a better distribution at other points along the clamped edge.

It does not seem reasonable that the maximum moment would actually be two per cent less. As may be seen from Table 3, noting that $M_x = U$ at point 8 because $\partial^2 w/\partial y^2 = 0$, the values of maximum deflection and moment of solution one are somewhat less than the values presented by Holl. This observation, plus the fact that the deflections at points B and C are in the wrong direction, lead to the conclusion that the finite difference solution to the problem is not very accurate for the net size used.

A better finite difference solution, with the same net size, is one in which the deflection at net point E, w_E, is not assumed equal to zero, as was assumed by Holl and solution one. The last equation of group [6] then becomes

$$(2w_B + w_E)N = - U_a \lambda^2$$

and the last equation of group [8] becomes

$$\frac{U_F - U_1}{2\lambda} + \frac{N(1 - \nu)}{2\lambda^3}(2w_F - w_G - 2w_1 + w_a - w_E) = 0$$

The additional equation required to make a system of forty equations in forty unknowns is obtained by setting My equal to zero at net point A. The equations of Holl's group [7] represent this condition, or the similar condition involving Mx, at the other boundary net points. The finite difference equation representing this condition at point A is as follows:

$$N(1 - \nu)2w_B = -U_a\lambda^2$$

With employment of the same constants as for solution one, the solution to this system of equations was obtained using a digital computer. The solution and the check on the accuracy are shown on pages 32, and 33. Coefficients calculated from this solution and corresponding to those of Holl are shown as solution two in Table 3 on page 68. It may be seen that the values of the coefficients for maximum deflection and moment, which is equal to U_8 as before, have increased and the coefficients for deflection at points B and C have become less negative. These facts indicate that solution two is better than solution one, but still not very close to what might be expected by comparison with the values of MacGregor's infinitely wide plate, and from the fact that all deflection coefficients should logically be positive.

Holl's unstated assumption that w_E is zero can be seen to be justified only to the extent that w_E is small compared to most of the other net point deflections, and the assumption that it is zero has only a relatively small effect on the values of the other unknowns.

CHAPTER III

THEORETICAL DEVELOPMENT

The expression used for the second derivative, in the application of the method of finite differences, is identical to the expression for the second derivative at any point on a parabola in terms of the ordinates of the parabola at the point in question and an equally spaced point on either side. This statement is also true of a third order polynomial, but not of a fourth or higher order polynomial. Proof of this statement for the parabola and third order polynomial may be found on pages 35 and 36 of the appendix.

Therefore, any problem involving only differential equations which are second derivatives of second or third order polynomials can be solved exactly by the method of finite differences. Net size will not, then, affect the accuracy of the values of the unknowns obtained. See, for instance, Timoshenko's (8) example of the uniformly loaded, long, rectangular plate. The differential equations involved are

$$\frac{\partial^2 M}{\partial x^2} = -q, \qquad \frac{\partial^2 w}{\partial x^2} = -\frac{M}{D}$$

The solution for the moment equation yields the second order polynomial

$$M = \frac{qa}{2}x - \frac{qx^2}{2}$$

and the finite difference solution for the values of moment are therefore exact, as shown. The deflection equation is the fourth order polynomial

$$w = \frac{qx}{24D}(a^3 - 2ax^2 + x^3)$$

and the finite difference solution for the values of w is therefore only approximate, as shown.

The fact that the accuracy of the finite difference solution of this problem for values of bending moment is not affected by net size can be illustrated as follows. Let the net size of the problem be four times as large, i.e. $\Delta x = a/2$, and let M_4 again be the moment at the middle of the span and M_0 the moment at the ends. The finite difference equation for

$$\Delta^2 M/\Delta x^2 = -q$$

is

$$M_0 - 2M_4 + M_0 = -2M_4 = -q\frac{a^2}{4}$$

The value of M_4 is therefore $qa^2/8$, which is identical with the value shown on page 183 of Timoshenko's (8) example.

The exact answer for w at the net points, as well as for M, can be obtained using the derivatives of a fourth or higher order polynomial, since the known solution for w is a fourth order polynomial. A symmetrical sixth order polynomial, for instance, will yield the exact values of w and M at the middle and at two net points on either side. Figure 2 illustrates the location of the net points, and the coefficients of the polynomial are calculated on pages 37 through 40 of the appendix, and are shown with the axes shifted and with a lowest common denominator in Table 5, page 70. The polynomial equations at net points 3, 2 and 1 for the equation

$$\partial^2 M/\partial x^2 = -q$$

are

$$-245/90\ M_3 + 270/90\ M_2 - 27/90\ M_1 = -qa^2/36$$
$$200/180\ M_3 - 405/180\ M_2 + 216/180\ M_1 = -qa^2/36$$
$$235/90\ M_3 - 270/90\ M_2 - 27/90\ M_1 = -qa^2/36$$

The solution to this system of equations is

$$M_1 = 5/72\ qa^2,\ M_2 = 1/9\ qa^2,\ M_3 = 1/8\ qa^2$$

The polynomial equations at net points 3, 2 and 1 for the equation

$$\partial^2 w/\partial x^2 = -M/D$$

are

$$-245/90\ w_3 + 270/90\ w_2 - 27/90\ w_1 = -(a^2/36)(qa^2/8D)$$
$$200/180\ w_3 - 405/180\ w_2 + 216/180\ w_1 = -(a^2/36)(qa^2/9D)$$
$$235/90\ w_3 - 270/90\ w_2 - 27/90\ w_1 = -(a^2/36)(5\ qa^2/72D)$$

The solution to this system of equations is

$$w_1 = 405\ qa^4/(416)(144\ D),\ w_2 = 97\ qa^4/729(12\ D),$$
$$w_3 = 5\ qa^4/384\ D$$

Evaluating the equation for w shown on page 8 at x equal to a/2, the value of deflection at the middle obtained is the same as that shown above for w_3. The value of 53.3 N for w_4, the midpoint deflection, shown on page 184 of Timoshenko's (8) example is equivalent to the $(5/384)\ qa^4/D$ value. The value of M_3, the moment at the midpoint obtained above, can be seen to be the same exact value as previously obtained. The values of M and w at x equals a/6 and a/3 are also exact, as may be verified from the respective equations.

The polynomial method of approximate solution will yield more accurate answers than the finite difference method for a given net size for all problems which have for a solution a polynomial of fourth order or higher, as shown on the previous pages, and also for problems the solution to which is not a polynomial of finite order n, as will be shown in the rest of this chapter.

The solution to the problem of the simply supported, uniformly loaded plate is of this latter type. Equation 126 in Timoshenko (9) is the "exact" solution for the deflection at any point on the plate, assuming small deflections. The finite difference approximate solution of this problem with a net size of a/4 is shown in Timoshenko (10). The partial differential equation for plate deflection is shown as equation 101 in Timoshenko (11), and this equation written as two equations, by defining M as shown on page 20, is also shown (12). These equations are reproduced on pages 40 and 41 of the appendix. For any plate problem, these equations must be satisfied at interior plate points. For the finite difference solution of the simply supported, uniformly loaded plate problem, these equations must be satisfied at interior net points, and M and w must be zero at the boundaries.

The finite difference solution with a net size of a/6 is more accurate. Figure 3 shows the coordinates and designation of the net points. Pages 40 and 41 of the appendix show the finite difference equations written at the various net points, giving a system of twelve equations in twelve unknowns. Assigning the values indicated to q, a and D, the solution shown on page 41 was obtained using a digital computer.

The polynomial solution with the same net size of a/6 is more accurate yet. This solution requires coefficients for a symmetrical sixth order polynomial, which is the same as that used previously on page 9 for the example of the uniformly loaded, long rectangular plate. Pages 42 and 43 of the appendix show the polynomial difference equations at the various net points. The same values for the quantities q, a and D as used in the finite difference solution were substituted in these equations and solution by digital computer was again obtained, with the results shown on page 43.

The value of maximum deflection, computed more accurately than the value given by Timoshenko (9), is as follows:

$$\begin{aligned} w_{max.} &= \frac{5}{384}\frac{qa^4}{D} - \frac{4}{\pi^5}\frac{qa^4}{D}(0.68562 - .00025 + \ldots) \\ &= .0040623\,\frac{qa^4}{D} = .04436\,\frac{qa^4}{Eh^3} \end{aligned}$$

Table 4 shows the comparison of the three approximate values for maximum deflection and moment with the more exact values obtained from

Timoshenko (9); it can be seen that the polynomial solution is the most accurate of the three approximations.

CHAPTER IV

POLYNOMIAL SOLUTION OF CANTILEVER PLATE PROBLEM

With the same net size, a polynomial solution of the problem of the bending of a cantilever plate by a concentrated edge load can be expected to be more accurate than the finite difference solution as attempted by Holl (1) and as carried out in Chapter II. The coefficients of a symmetrical sixth order and tenth order polynomial are required in order to be able to write the polynomial equivalents of the differential equations at the various net points. The coefficients of the sixth order polynomial have been used in Chapters II and III and those of the tenth order polynomial are calculated on pages 44 through 49 of the appendix.

It can be seen that the coefficients for the symmetrical sixth and tenth order polynomials on pages 39, 48 and 49 have the symmetry and anti-symmetry that would be expected for the first and second derivatives of symmetrical functions and that the first derivative is zero at the value of x about which the function is symmetrical.

The coefficients are shown in Tables 5 and 6 with the symmetry shifted to the origin, about which the symmetry exists for the problem under consideration, and with all coefficients expressed with the lowest common denominator at any given value of x. This facilitates the writing of the system of equations.

The polynomial difference equations are derived on pages 50 through 62 of the appendix. Each equation corresponds to the noted equation of

Holl's system of equations with w_E not assumed zero, and a fortieth equation is added on page 57 after the seven equations corresponding to Holl's group [7]. This equation, which sets My equal to zero at net point A, is similar to the others of group [7]. See page 55. These equations, then, correspond to those of the better finite difference solution mentioned on pages 5 through 6 . A Poisson's ratio of three tenths is again used, and the same constants as were used in Chapter II are substituted in the equations to permit solution by digital computer. The quantities w_8, w_7, w_6, w_5, w_A and U_C are shown in the equations as they would be in a general case, but for the specific solution of this cantilever plate problem, they are set equal to zero; the deflections at the wall are zero for the coordinate system chosen, and U_C is proportional to the sum of the quantities Mx and My, as shown on page 20, with Timoshenko's notation, both of which are zero at the corner point C.

The polynomial difference equations corresponding to Holl's group [8] are somewhat more difficult to derive than the others. As shown on pages 23 and 24 it is necessary to write a first partial derivative of a second partial derivative. This is accomplished on pages 57 through 62 by treating the entire expression for second derivative at a net point as the quantities U and w have been previously treated, and applying the proper coefficients for the first derivatives to the entire expressions for the second derivatives.

The system of forty equations in forty unknowns thus obtained was solved by use of a digital computer, as before. The values of the unknowns, the values computed by substituting the values of the unknowns in the equations, and the right hand sides of the equations are shown on pages 63 and 64 in floating point form, exactly as printed out by the computer. It can be seen that the solution obtained is not as exact as the other solutions obtained with the computer; this is due to the fact that the equations contain coefficients with as many as eight significant figures, which is the maximum number the computer can be given, and the internal calculations of the computer, for the program used, are only accurate to eight or nine significant figures in the decimal system. The maximum error of the solution, comparing the error in the right hand side of an equation with the minimum absolute value of the numbers, coefficients times values of the unknowns, which add together to give the right hand side, is about 7 1/2 per cent for the equation corresponding to the first of Holl's group [8], based on the number involving w_H. The next largest error, computed in this manner, is about 2 1/2 per cent for the next equation, again based on the number involving w_H. No other equation has an error, computed in this manner, of as much as one percent, and the error of most is much less.

The values of the unknowns, converted to the coefficient form of Holl's results, are presented as solution three in Table 3, page 68. It can be seen that the maximum value of w_d of .24683 Pa^2/N exceeds MacGregor's (2) results by 47 per cent, and that the value of maximum Mx, at point 8, which is equal to U_8, exceeds MacGregor's results by 33 per cent; also, the deflections w_C and w_B are in the expected direction. These results seem more reasonable than the finite difference method results discussed on pages 5 and 6, and seem to indicate that for this problem, as with the problems in Chapter III, the polynomial solution is more accurate than the finite difference solution for a given net size.

CHAPTER V

CONCLUSIONS

The method of finite differences, with the net size used by Holl and in Chapter II, does not give reasonably accurate results for the problem of the elastic cantilever plate with a width twice the length and a concentrated load applied at the center of the free edge.

The method of solution by polynomial gives a more accurate solution of elastic plate problems, for a given net size, than the method of finite differences except for certain simple problems in which the two methods are equivalent.

The method of solution by polynomials, as applied to the problem of the elastic cantilever plate mentioned in the first paragraph above, yields a more accurate solution than the finite difference method for the net size used.

CHAPTER VI

RECOMMENDATIONS

The method of approximate solution by polynomials can be used to obtain solutions to any desired degree of accuracy for problems involving ordinary or partial differential equations. This can be accomplished by determining the coefficients for polynomials with as high an order as desired, and using them to form a system of equations as was done in Chapters III and IV. It is only recently, with the development of high speed computers, that this method has become practical to use because of the large size of the system of equations, and the large number of significant figures in the coefficients, required for the solution of a problem not more easily obtained in some other way. Future development of even faster digital computers than the IBM 704, and programming of them to obtain the highest possible accuracy in the solution of systems of linear equations, will permit the use of polynomials of increasingly higher order in obtaining solutions to problems.

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2. MacGregor, C. W., "Deflection of a Long Helical Gear Tooth", *Mechanical Engineering*, Vol. 57, 1935, p. 225.

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5. *Ibid.*, p. 468.

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7. Timoshenko, S., *Theory of Plates and Shells*, 1st ed. New York: McGraw-Hill Book Company, Inc., 1940, pp. 180-187.

8. Timoshenko, *op. cit.*, pp. 180-184.

9. *Ibid.*, pp. 128-129.

10. *Ibid.*, pp. 184-187.

11. *Ibid.*, p. 88.

12. *Ibid.*, p. 100.

13. *Ibid.*, pp. 89-90.

14. Ibid., p. 133.

APPENDIX

The equations shown below are the general partial differential equations, as shown in Timoshenko (11) (12) for bending of rectangular elastic plates by lateral loads.

$$Mx = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right), \quad My = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right)$$

$$Mxy = -Myx = D(1 - \nu) \frac{\partial^2 w}{\partial x \partial y}$$

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D}$$

If, for a particular case, a solution of the equation immediately above can be found that satisfies the conditions at the boundary of the plate, the bending and twisting moments can be calculated from the three preceding it.

The fourth order equation above can be replaced by two equations of the second order by writing the fourth order equation in the following form:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial^2 w}{x^2} + \frac{\partial^2 w}{\partial y^2}\right) = \frac{q}{D}$$

and observe that by adding the two expressions above for bending moment we have

$$M_x + M_y = -D(1 + \nu)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

Introducing the notation

$$M = \frac{M_x + M_y}{1 + \nu} = -D\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \quad ,$$

the following equations can be written, which represent the preceding two equations:

$$\frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2} = -q, \; \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = -\frac{M}{D}$$

The quantity M is zero at a simply supported boundary because, for instance at an edge x = constant, the deflection is zero all along the edge and $M_x = 0$. Since the deflection is zero along the edge, $\partial^2 w/\partial y^2 = 0$, and with this being true, the only way M_x can equal zero, as may be seen from the equation for M_x, is for $\partial^2 w/\partial x^2$ to also be zero at the edge. This being true, the quantity M must also be zero.

The system of equations appearing in the article by D. C. Holl is shown below. The equations at interior net points for the last equations shown above are

$$4U_1 - U_2 - U_a - U_B - U_5 = 0 \qquad [1]$$
$$4U_2 - U_3 - U_1 - U_b - U_6 = 0$$
$$4U_3 - U_4 - U_2 - U_c - U_7 = 0$$
$$4U_4 - U_3 - U_3 - U_d - U_8 = 0$$

$$\begin{aligned}
4U_a - U_b - U_1 - U_\alpha &= 0 \\
4U_b - U_c - U_a - U_2 - U_\beta &= 0 \\
4U_c - U_d - U_b - U_3 - U\gamma &= 0 \\
4U_d - U_c - U_c - U_4 - U_\delta &= 0 \\
4U_B - U_1 - U_F - U_A &= 0 \\
- U_a - U_G - U_B - U_D &= 0
\end{aligned} \qquad [2]$$

$$\begin{aligned}
N(4w_1 - w_2 - w_B - w_a) &= U_1\lambda^2 \\
N(4w_2 - w_3 - w_1 - w_b) &= U_2\lambda^2 \\
N(4w_3 - w_4 - w_2 - w_c) &= U_3\lambda^2 \\
N(4w_4 - w_3 - w_3 - w_d) &= U_4\lambda^2
\end{aligned} \qquad [3]$$

$$\begin{aligned}
N(4w_a - w_b - w_C - w_1 - w_\alpha) &= U_a\lambda^2 \\
N(4w_b - w_c - w_a - w_2 - w_\beta) &= U_b\lambda^2 \\
N(4w_c - w_d - w_b - w_3 - w_\gamma) &= U_c\lambda^2 \\
N(4w_d - w_c - w_c - w_4 - w_\delta) &= U_d\lambda^2 \\
N(4w_B - w_1 - w_F - w_C \quad\quad) &= U_B\lambda^2 \\
N(4w_C - w_a - w_G - w_D - w_F) &= 0
\end{aligned} \qquad [4]$$

The corner condition, corresponding to the twisting moment being zero, which Holl later states to be a redundant relation, being equivalent to two other equations, is as follows:

$$w_\alpha + w_F - w_1 - w_H = 0 \qquad [5]$$

The equations at the wall net points for the third equation shown on page 20 are

$$
\begin{aligned}
2w_1N &= -U_5\lambda^2 \\
2w_2N &= -U_6\lambda^2 \\
2w_3N &= -U_7\lambda^2 \\
2w_4N &= -U_8\lambda^2 \\
2w_BN &= -U_A\lambda^2
\end{aligned}
\qquad [6]
$$

Along the free edge $x = a$, the two relations

$$N\Delta^2 w = -U \text{ and } Mx = -N\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) = 0 \text{ yield}$$

$$N(1 - \nu)\,\frac{\partial^2 w}{\partial y^2} = -U_{\text{edge}}.$$

Similarly along the free edges $y = \pm 2a$

$$N(1 - \nu)\,\frac{\partial^2 w}{\partial x^2} = -U_{\text{edge}}.$$

These conditions along the edges $x = a$, and $y = \pm 2a$ yield the following:

$$
\begin{aligned}
N(1-\nu)(2w_a - w_b - w_C) &= U_a\lambda^2 \\
N(1-\nu)(2w_b - w_c - w_a) &= U_b\lambda^2 \\
N(1-\nu)(2w_c - w_d - w_b) &= U_c\lambda^2 \\
N(1-\nu)(2w_d - 2w_c) &= U_d\lambda^2 \\
N(1-\nu)(2w_B - w_C) &= U_B\lambda^2 \\
2w_C - w_a - w_G &= 0 \\
2w_C - w_D - w_B &= 0
\end{aligned}
\qquad [7]
$$

The last of Holl's equations represent the condition that the shear and twisting moment be zero at the free edge. The equations for shear at a free edge are

$$Qx = \frac{\partial Myx}{\partial y} + \frac{\partial Mx}{\partial x} = -D\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

$$Qy = \frac{\partial My}{\partial y} + \frac{\partial Mxy}{\partial x} = -D\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

as shown in Timoshenko (11). It is also shown in Timoshenko (13) that the joint requirement concerning twisting moment Mxy and shearing force Qx along a free edge x = a is

$$Vx = \left(Qx - \frac{\partial Mxy}{\partial y}\right)_{x\,=\,a} = 0$$

Substituting the equations for Qx above and for Mxy, as shown on page 19, the following results:

$$Vx = 0 = -D\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) - \frac{\partial}{\partial y}\left[D(1-\nu)\frac{\partial^2 w}{\partial x \partial y}\right]$$

This may be rewritten, using Holl's notation instead of Timoshenko's as follows:

$$\frac{\partial U}{\partial x} - N(1-\nu)\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial y^2}\right)$$

The corresponding equation may also be written for the edges y = ±2a. Holl's last group of equations represent this condition.

$$\frac{U_\delta - U_4}{2\lambda} + \frac{N(1-\nu)}{2\lambda^3}(2w_\delta - 2w_\gamma - 2w_4 - 2w_3) = \frac{P}{\lambda} \qquad [8]$$

$$\frac{U\lambda - U_3}{2\lambda} + \frac{N(1-\nu)}{2\lambda^3}(2w_\gamma - w_\delta - w_\beta - 2w_3 + w_4 + w_2) = 0$$

$$\frac{U_\beta - U_2}{2\lambda} + \frac{N(1-\nu)}{2\lambda^3}(2w_\beta - w_\gamma - w_\alpha - 2w_2 + w_3 + w_1) = 0$$

$$\frac{U_\alpha - U_1}{2\lambda} + \frac{N(1-\nu)}{2\lambda^3}(2w_\alpha - w_\beta - w_D - 2w_1 + w_2 + w_B) = 0 \qquad [8]$$

$$\frac{U_D - U_B}{2\lambda} + \frac{N(1-\nu)}{2\lambda^3}(2w_D - w_\alpha - w_H - 2w_B + w_1 + w_F) = 0$$

$$\frac{U_G - U_a}{2\lambda} + \frac{N(1-\nu)}{2\lambda^3}(2w_G - w_F - w_H - 2w_a + w_1 + w_\alpha) = 0$$

$$\frac{U_F - U_1}{2\lambda} + \frac{N(1-\nu)}{2\lambda^3}(2w_F - w_G - 2w_1 + w_a \qquad) = 0$$

The twenty six equations mentioned on page 3 are satisfied by the thirty-two unknowns presented by Holl to the extent shown below. The coefficients for U in groups [3], [6] and [7] are divided by four because $\lambda = a/2$, and the coeffients for w are in terms of a, as shown in Table 3. Poisson's ratip equals three tenths. The w_F term in the last equation of group [4] has been replaced by w_B to correct one of Holl's apparent errors. Note that substitution of w_F would cause the equation to be not satisfied.

$$4(-.08634) + .13089 + .039 + .04948 + .12600 = .00001$$
$$4(-.13089) + .15847 + .08634 + .05205 + .22672 = .00002$$
$$4(-.15847) + .13091 + .13089 - .00143 + .37352 = .00001 \qquad [1]$$
$$4(-.13091) + .15847 + .15847 - .29002 + .49672 = 0.$$

$$4(.01575) - .02834 - .00624 - .04993 + .08634/4 = .000075$$
$$4(.02834) - .04669 - .01575 - .08364 + .13089/4 = .0000025$$
$$4(.04669) - .06209 - .02834 - .13594 + .15847/4 = .0000075 \; [3]$$
$$4(.06209) - .04669 - .04669 - .18773 + .13091/4 = -.0000225$$

$$4(.04993) - .08364 - .03015 - .01575 - .07993 + .039/4 = 0 \qquad [4]$$

$$4(.08364) - .13594 - .04993 - .02834 - .13336 + .05205/4 = -.0000025$$

$$4(.13594) - .18773 - .08364 - .04669 - .22534 - .00143/4 = .0000025$$

$$4(.18773) - .13594 - .13594 - .06209 - .34445 - .29002/4 = -.000005$$

$$4(.00624) - .01575 + .00857 - .03015 + .04948/4 = 0$$

$$4(.03015) - .04993 - .01037 - .05406 - .00624 = 0$$

$$2(.01575) - .12600/4 = 0.$$

$$2(.02834) - .22672/4 = 0.$$

$$2(.04669) - .37352/4 = 0. \qquad [6]$$

$$2(.06209) - .49672/4 = 0.$$

$$2(.00624) - .04992/4 = 0.$$

$$1.4(.04993) - .7(.08364) - .7(.03015) + .039/4 = -.000001$$

$$1.4(.08364) - .7(.13594) - .7(.04993) + .05205/4 = -.0000005$$

$$1.4(.13594) - .7(.18773) - .7(.08364) - .00143/4 = -.0000005 \qquad [7]$$

$$1.4(.18773) - 1.4(.13594) - .29002/4 = .000001$$

$$1.4(.00624) - .7(.03015) + .04948/4 = .00001$$

$$2(.03015) - .04993 - .01037 = 0.$$

$$2(.03015) - .05406 - .00624 = 0.$$

The seven unknowns not presented by Holl (1) in his article can be solved for by using his equations of group [8], each of which contains only one of the unknowns. The first equation is written with $a + 2w_3$ instead of $a - 2w_3$ to correct another apparent error. Again, substitution of $a/2$ for λ is needed to make the equations dimensionally correct when the coefficients are substituted.

$$U_\delta = (2.0 + U_4 - 5.6w_\delta + 5.6w_\gamma + 5.6w_4 - 5.6w_3)$$

$$= [2.0 + (-.13091) - 5.6(.34445) + 5.6(.22534) + 5.6(.06209) - 5.6(.04669)]P$$

$$U_\gamma = U_3 - 5.6w_\gamma + 2.8w_\delta + 2.8w_\beta + 5.6w_3 - 2.8w_4 - 2.8w_2$$
$$= \Big[-.15847 - 5.6(.22534) + 2.8(.34445) + 2.8(.13336) + 5.6(.04669) - 2.8(.06209) - 2.8(.02834)\Big] P$$

$$U_\beta = U_2 - 5.6w_\beta + 2.8w_\gamma + 2.8w_\alpha + 5.6w_2 - 2.8w_3 - 2.8w_1$$
$$= \Big[-.13089 - 5.6(.13336) + 2.8(.22534) + 2.8(.07993) + 5.6(.02834) - 2.8(.04669) - 2.8(.01575)\Big] P$$

$$U_\alpha = U_1 - 5.6w_\alpha + 2.8w_\beta + 2.8w_D + 5.6w_1 - 2.8w_2 - 2.8w_B$$
$$= \Big[-.08634 - 5.6(.07993) + 2.8(.13336) + 2.8(.05406) + 5.6(.01575) - 2.8(.02834) - 2.8(.00624)\Big] P$$

$$U_D = U_B - 5.6w_D + 2.8w_\alpha + 2.8w_H + 5.6w_B - 2.8w_1 - 2.8w_F$$
$$= \Big[-.04948 - 5.6(.05406) + 2.8(.07993) + 2.8(.07275) + 5.6(.00624) - 2.8(.01575) - 2.8(-.00857)\Big] P$$

$$U_G = U_a - 5.6w_G + 2.8w_F + 2.8w_H + 5.6w_a - 2.8w_1 - 2.8w_\alpha$$
$$= \Big[-.03900 - 5.6(.01037) + 2.8(-.00857) + 2.8(.07275) + 5.6(.04993) - 2.8(.01575) - 2.8(.07993)\Big] P$$

$$U_F = U_1 - 5.6w_F + 2.8w_G + 5.6w_1 - 2.8w_a$$
$$= \Big[-.08634 - 5.6(-.00857) + 2.8(.01037) + 5.6(.01575) - 2.8(.04993)\Big] P$$

$$U_\delta = 1.288314P,\ U_\gamma = -.074246P,\ U_\beta = -.039078P,$$
$$U_\alpha = -017796P$$
$$U_D = .090128P,\ U_G = .094336P,\ U_F = -.060916P$$

With values for all thirty-nine unknowns available, the accuracy with which the remaining six equations of group [2] are satisfied is shown below.

$$4U_a - U_b - U_1 - U_\alpha$$
$$= \left[4(-.039) + .05205 + .08634 - (-.017796)\right] P = .000186P$$

$$4U_b - U_c - U_a - U_2 - U_\beta$$
$$= \left[4(-.05205) - .00143 + .039 + .13089 - (-.039078)\right] P = -.000662P$$

$$4U_c - U_d - U_b - U_3 - U_\gamma$$
$$= \left[4(.00143) - .29002 + .05205 + .15847 - (-.074246)\right] P = .000466P$$

$$4U_d - 2U_c - U_4 - U_\delta$$
$$= \left[4(.29002) - 2(.00143) + .13091 - 1.288314\right] P = -.000184P$$

$$4U_B - U_1 - U_F - U_A$$
$$= \left[4(-.04948) + .08634 - (-.060916) + .04992\right] P = -.000744P$$

$$-U_a - U_G - U_B - U_D$$
$$= \left[-(-.039) - .094336 - (-.04948) - .090128\right] P = -.095984P$$

The system of equations mentioned on page 4 is as follows:

$$4U_1 - U_2 - U_5 - U_a - U_B = 0$$

$$-U_1 + 4U_2 - U_3 - U_6 - U_b = 0$$

$$-U_2 + 4U_3 - U_4 - U_7 - U_c = 0 \qquad [1]$$

$$-2U_3 + 4U_4 - U_8 - U_d = 0$$

$$-U_1 + 4U_a - U_b - U_\alpha = 0$$

$$-U_2 - U_a + 4U_b - U_c - U_\beta = 0$$

$$-U_3 - U_b + 4U_c - U_d - U_\gamma = 0 \qquad [2]$$

$$-U_4 - 2U_c + 4U_d - U_\delta = 0$$

$$-U_1 - U_A + 4U_B - U_F = 0$$

$$-U_a - U_B - U_D - U_G = 0$$

$$-\ .001\ U_1 + 4w_1 - w_2 - w_a - w_B = 0$$

$$-\ .001\ U_2 - w_1 + 4w_2 - w_3 - w_b = 0$$

$$-\ .001\ U_3 - w_2 + 4w_3 - w_4 - w_c = 0 \qquad [3]$$

$$-\ .001\ U_4 - 2w_3 + 4w_4 - w_d = 0$$

$$-\ .001\ U_a - w_1 + 4w_a - w_b - w_C - w_\alpha = 0$$

$$-\ .001\ U_b - w_2 - w_a + 4w_b - w_c - w_\beta = 0$$

$$-\ .001\ U_c - w_3 - w_b + 4w_c - w_d - w_\gamma = 0 \qquad [4]$$

$$-\ .001\ U_d - w_4 - 2w_c + 4w_d - w_\delta = 0$$

$$-\ .001\ U_B - w_1 + 4w_B - w_C - w_F = 0$$

$$-\ w_a - w_B + 4w_C - w_D - w_G = 0$$

$$.001\ U_5 + 2w_1 = 0$$
$$.001\ U_6 + 2w_2 = 0$$
$$.001\ U_7 + 2w_3 = 0 \qquad [6]$$
$$.001\ U_8 + 2w_4 = 0$$
$$.001\ U_A + 2w_B = 0$$

$$-\ .001\ U_a + 1.4w_a - .7w_b - .7w_C = 0$$
$$-\ .001\ U_b - .7w_a + 1.4w_b - .7w_c = 0$$
$$-\ .001\ U_c - .7w_b + 1.4w_c - .7w_d = 0 \qquad [7]$$
$$-\ .001\ U_d - 1.4w_c + 1.4w_d = 0$$
$$-\ .001\ U_B + 1.4w_B - .7w_C = 0$$
$$- w_a + 2w_C - w_G = 0$$
$$- w_B + 2w_C - w_D = 0$$

$$- U_4 + U_\delta + 1400\ w_3 - 1400\ w_4 - 1400\ w_\gamma + 1400\ w_\delta = 500$$
$$- U_3 + U_\gamma + 700\ w_2 - 1400\ w_3 + 700\ w_4 - 700\ w_\beta + 1400\ w_\gamma - 700\ w_\delta = 0$$
$$- U_2 + U_\beta + 700\ w_1 - 1400\ w_2 + 700\ w_3 - 700\ w_\alpha + 1400\ w_\beta - 700\ w_\gamma = 0 \qquad [8]$$
$$- U_1 + U_\alpha - 1400\ w_1 + 700\ w_2 + 700\ w_B - 700\ w_D + 1400\ w_\alpha - 700\ w_\beta = 0$$
$$- U_B + U_D + 700\ w_1 - 1400\ w_B + 1400\ w_D - 700\ w_H + 700\ w_F - 700\ w_\alpha = 0$$
$$- U_a + U_G + 700\ w_1 - 1400\ w_a + 1400\ w_G - 700\ w_H - 700\ w_F - 700\ w_\alpha = 0$$
$$- U_1 + U_F - 1400\ w_1 + 700\ w_a - 700\ w_G + 1400\ w_F = 0$$

The values of the unknowns, and the numbers obtained by substituting the values of the unknowns into the equations as computed by the digital computer, plus the magnitude of the right hand sides of the equations, for the system of equations mentioned on page 4 and in the order presented on pages 28 and 29, are shown below and on the next page in floating point form.

UNKNOWNS	VALUES OF UNKNOWNS	CHECK	RHS OF EQUATIONS
U_1	2-12496263	-5-70395630	0
U_2	2-26191676	-5 43926479	0
U_3	2-35211816	-4-11510103	0
U_4	2-29075010	-5-43215907	0
U_5	2-14237463	-5 19588319	0
U_6	2-47803558	-5 92679926	0
U_7	2-88810905	-4 13682171	0
U_8	3-12090341	-5-56319393	0
U_a	1-59158299	-5 86912282	0
U_b	1-92550478	-5 49056563	0
U_c	1 32303529	-7-45199367	0
U_d	2 75027033	-7-74809396	0
U_A	2 16442343	-7-31305436	0
U_B	1-36400817	-7 18515465	0
U_D	1 52331052	-7-48850379	0
U_G	2-18506428	-7 33012482	0
U_F	1 43228120	-7 17084078	0
U_α	1-19120099	-7 74505806	0
U_β	1-81430496	-7 42628982	0
U_γ	2-17638739	-8-76705424	0
U_δ	3 32272244	-8-67737153	0
W_1	-2 71187314	-8-31350956	0

UNKNOWN	VALUES OF UNKNOWNS	CHECK	RHS OF EQUATIONS
w_2	-1 23901784	-8 16404593	0
w_3	-1 44405452	-7 31880093	0
w_4	-1 60451693	-8-79566590	0
w_a	-1 25290596	-8-18345392	0
w_b	-1 70274614	-8 23748011	0
w_c	12848014	-8 15268229	0
w_d	18207088	-7 12348220	0
w_B	-2-82211699	-8 97230788	0
w_C	-1-11242242	-7-11896999	0
w_D	-1-14263293	-8 17262873	0
w_G	-1-25121131	3 49999996	500
w_H	-1-47775044	-4 13351440	0
w_F	-1-58337697	-4 24872480	0
w_α	-1 40927130	-6 10238382	0
w_β	11268100	-4-17865437	0
w_γ	21393928	-4-19152696	0
w_δ	33584451	-4 15679868	0

The equations of the system mentioned on pages 5 and 6 which are different from those shown on pages 28 and 29 are shown below. The last equation of group [6] becomes

$$.001\ U_A + 2w_B + w_E = 0$$

The last equation of group [8] becomes

$$- U_1 + U_F - 1400\ w_1 + 700\ w_a - 700\ w_G + 1400\ w_F - 700\ w_E = 0$$

The additional equation for net point A, corresponding to the equations of group [7] for the other boundary net points is

$$- .001\, U_A - 1.4\, w_B = 0$$

The values of the unknowns and the check as computed by the digital computer, and the right hand side of the equations for reference purposes, are shown below and on the following pages. The additional equation is the thirty-third, right after the equations corresponding to Holl's group [7].

UNKNOWNS	VALUES OF UNKNOWNS	CHECK	RHS OF EQUATIONS
U_1	2-14245270	-5 36725785	0
U_2	2-27437590	-5-39977387	0
U_3	2-36055078	-4-10772570	0
U_4	2-29775561	-5-43660103	0
U_5	2-17461592	-5-17016139	0
U_6	2-49488473	-5 75410983	0
U_7	2-89688242	-5-46941235	0
U_8	3-12153920	-4-37592189	0
U_a	1-66425637	-5 31542516	0
U_b	1-99615255	-6-14253348	0
U_c	1 26811098	-7-36429175	0
U_d	2 74547148	-7-15305231	0
U_A	1 77953034	-7-11886520	0
U_B	1-54393367	-8-11377099	0
U_D	1 62815938	-7-41973406	0

UNKNOWNS	VALUES OF UNKNOWNS	CHECK	RHS OF EQUATIONS
U_G	2-15307385	-8-80507224	0
U_F	1 58003098	-7 85698508	0
U_α	1-23634625	-7-22228185	0
U_β	1-84470657	-8 41607788	0
U_γ	2-17806077	-7 22003702	0
U_δ	3 32260194	-8 99826967	0
w_1	-2 87307951	-10-31981051	0
w_2	-1 24744238	-9-31895630	0
w_3	-1 44844120	-9 41829079	0
w_4	-1 60769588	-8-70431269	0
w_a	-1 29992288	-9-57626553	0
w_b	-1 72839628	-8 20936163	0
w_c	12991772	-7 11438976	0
w_d	18316568	-7-13193350	0
w_B	-2-55680760	-9-17161339	0
w_C	-2-33656721	-8 26236142	0
w_D	-2-11632703	-8 68834585	0
w_G	-1-22198090	-8 38389680	0
w_H	-1-36723636	3 50000002	500.
w_F	-1-36320465	-4-50023724	0
w_α	-1 48406969	-4 51150259	0
w_β	11666579	-4-17137723	0
w_γ	21614038	-5 12290482	0
w_δ	33751053	-4 19229124	0
w_E	-2 33408478	-4-22143125	0

GENERAL DERIVATION OF POLYNOMIAL COEFFICIENTS

Let y be a function of x and let y be tabulated at various values of x, with x varying by the constant value λ so that

$$x_1 = x_o + \lambda, \quad x_2 = x_1 + \lambda, \quad \text{etc.}$$

If y_o corresponds to x_o, y_1 to x_1, and so on, successive differences in y, and successive differences of those differences can be denoted as shown in the table below, where a_1 equals $(y_1 - y_o)$, a_2 equals $(y_2 - y_1)$, $b_1 = (a_2 - a_1)$, etc.

$$\begin{array}{cccccc}
x_o & y_o & & & & \\
 & & a_1 & & & \\
x_1 & y_1 & & b_1 & & \\
 & & a_2 & & c_2 & \\
x_2 & y_2 & & b_2 & & d_2 \\
 & & a_3 & & c_3 & \\
x_3 & y_3 & & b_3 & & \\
 & & a_4 & & & \\
x_4 & y_4 & & & &
\end{array}$$

In order to get a formula for y in terms of x such that $y = y_o$ when $x = x_o$, $y = y_1$ when $x = x_1$, etc., it can be written that

$$y = A_o + (x - x_o)A_1 + (x - x_o)(x - x_1)A_2 + (x - x_o)(x - x_1)(x - x_2)\, A_3 + \ldots .$$

where

$$A_o = y_o, \quad A_1 = a_1/\lambda, \quad A_2 = b_1/2!\,\lambda^2, \quad A_3 = c_2/3!\,\lambda^3, \text{ etc.}$$

The proofs of the statements made on page 7 concerning the second derivative of a second and third order polynomial are shown below and on the next page for values of x spaced a distance λ apart.

$$
\begin{array}{llll}
x & y & a & b \\
x_{n-1} & y_{n-1} & & \\
 & & y_n - y_{n-1} & \\
x_n & y_n & & y_{n+1} - 2y_n + y_{n-1} \\
 & & y_{n+1} - y_n & \\
x_{n+1} & y_{n+1} & &
\end{array}
$$

$$A_o = y_{n-1}, \quad A_1 = (y_n - y_{n-1})/\lambda, \quad A_2 = (y_{n+1} - 2y_n + y_{n-1})/2\lambda^2$$

$$y = y_{n-1} + (x - x_{n-1})\frac{y_n - y_{n-1}}{\lambda} + (x - x_{n-1})(x - x_n)\frac{y_{n+1} - 2y_n + y_{n-1}}{2\lambda^2}$$

$$\frac{dy}{dx} = \frac{y_n - y_{n-1}}{\lambda} - (x_n + x_{n-1})\frac{y_{n+1} - 2y_n + y_{n-1}}{2\lambda^2} + 2x\frac{y_{n+1} - 2y_n + y_{n-1}}{2\lambda^2}$$

$$\frac{d^2y}{dx^2} = \frac{y_{n+1} - 2y_n + y_{n-1}}{\lambda^2}$$

$$
\begin{array}{lllll}
x & y & a & b & c \\
x_{n-1} & y_{n-1} & & & \\
 & & y_n - y_{n-1} & & \\
x_n & y_n & & y_{n+1} - 2y_n + y_{n-1} & \\
 & & y_{n+1} - y_n & & y_{n+2} - 3y_{n+1} + 3y_n - y_{n-1} \\
x_{n+1} & y_{n+1} & & y_{n+2} - 2y_{n+1} + y_n & \\
 & & y_{n+2} - y_{n+1} & & \\
x_{n+2} & y_{n+2} & & &
\end{array}
$$

A_o, A_1 and A_2 are the same as before and

$$A_3 = (y_{n+2} - 3y_{n+1} + 3y_n - y_{n-1})/6\lambda^3$$

$$y = y_{n-1} + (x - x_{n-1})\frac{y_n - y_{n-1}}{\lambda} + (x - x_{n-1})(x - x_n)\frac{y_{n+1} - 2y_n + y_{n-1}}{2\lambda^2}$$

$$+ (x - x_{n-1})(x - x_n)(x - x_{n+1})\frac{y_{n+2} - 3y_{n+1} + 3y_n - y_{n-1}}{6\lambda^3}$$

$$\frac{dy}{dx} = \frac{y_n - y_{n-1}}{\lambda} + (2x - x_n - x_{n-1})\frac{y_{n+1} - 2y_n + y_{n-1}}{2\lambda^2}$$

$$+ \left[3x^2 - 2x(x_{n-1} + x_n + x_{n+1}) + (x_n x_{n-1} + x_n x_{n+1} + x_{n-1}x_{n+1})\right] \cdot$$

$$\left[\frac{y_{n+2} - 3y_{n+1} + 3y_n - y_{n-1}}{6\lambda^3}\right]$$

$$\frac{d^2y}{dx^2} = \frac{y_{n+1} - 2y_n + y_{n-1}}{\lambda^2} + \left[6x - 2(x_{n-1} + x_n + x_{n+1})\right] \frac{y_{n+2} - 3y_{n+1} + 3y_n - y_{n-1}}{6\lambda^3}$$

Utilizing the relationship

$$x_{n-1} = (n - 1)\lambda, \quad x_n = n\lambda, \quad x_{n+1} = (n + 1)\lambda, \quad x_{n+2} = (n + 2)\lambda$$

$$\frac{d^2y}{dx^2} = \frac{y_{n+1} - 2y_n + y_{n-1}}{\lambda^2} + (6x - 6n\lambda)\frac{y_{n+2} - 3y_{n+1} + 3y_n - y_{n-1}}{6\lambda^3}$$

Evaluating this second derivative at $x = x_n = n\lambda$, and at $x = x_{n+1} = (n + 1)\lambda$,

$$\left(\frac{d^2y}{dx^2}\right)_{x_n} = \frac{1}{\lambda^2}(y_{n+1} - 2y_n + y_{n-1}), \left(\frac{d^2y}{dx^2}\right)_{x_{n+1}} = \frac{1}{\lambda^2}(y_{n+2} - 2y_{n+1} + y_n)$$

DERIVATION OF COEFFICIENTS OF A
SYMMETRICAL SIXTH ORDER POLYNOMIAL

The coefficients for a symmetrical sixth order polynomial can be arrived at in the same manner as that used on the previous pages for a second and third order polynomial. Letting y have the values y_3, y_2, y_1, y_o, y_1, y_2, y_3 at the values of x equal to 0, λ, 2λ, 3λ, 4λ, 5λ, 6λ, the equation y equals f(x) is derived as follows:

$$A_1 = \frac{y_2 - y_3}{\lambda}, \quad A_2 = \frac{y_4 - 2y_2 + y_3}{2\lambda^2}, \quad A_3 = \frac{y_o - 3y_1 + 3y_2 - y_3}{6\lambda^3}$$

$$A_4 = \frac{y_1 - 4y_o + 6y_1 - 4y_2 + y_3}{24\lambda^4}, \quad A_5 = \frac{y_2 - 5y_1 + 10y_o - 10y_1 + 5y_2 - y_3}{120\lambda^5}$$

$$A_6 = \frac{y_3 - 6y_2 + 15y_1 - 20y_o + 15y_1 - 6y_2 + y_3}{720\lambda^6}$$

$$y = y_3 + x\frac{y_2 - y_3}{\lambda} + (x^2 - \lambda x)\frac{y_1 - 2y_2 + y_3}{2\lambda^2} + (x^3 - 3\lambda x^2 + 2\lambda^2 x)\cdot$$

$$\frac{y_o - 3y_1 + 3y_2 - y_3}{6\lambda^3} + (x^4 - 6\lambda x^3 + 11\lambda^2 x^2 - 6\lambda^3 x)\frac{-4y_o + 7y_1 - 4y_2 + y_3}{24\lambda^4}$$

$$+ (x^5 - 10\lambda x^4 + 35\lambda^2 x^3 - 50\lambda^3 x^2 + 24\lambda^4 x)\frac{10y_o - 15y_1 + 6y_2 - y_3}{120\lambda^5}$$

$$+ (x^6 - 15\lambda x^5 + 85\lambda^2 x^4 - 225\lambda^3 x^3 + 274\lambda^4 x^2 - 120\lambda^5 x)\frac{-20y_o + 30y_1 - 12y_2 + 2y_3}{720\lambda^6}$$

This equation becomes, with algebraic manipulation,

$$y = y_3 + x\frac{400y_o - 675y_1 + 432y_2 - 157y_3}{60\lambda} + x^2\frac{-5080y_o + 8235y_1 - 4104y_2 + 949y_3}{360\lambda^2}$$

$$+ x^3 \frac{496y_o - 768y_1 + 336y_2 - 64y_3}{48\lambda^3} + x^4 \frac{-484y_o + 732y_1 + 300y_2 + 52y_3}{144\lambda^4}$$

$$+ x^5 \frac{360y_o - 540y_1 + 216y_2 - 36y_3}{720\lambda^5} + x^6 \frac{-20y_o + 30y_1 - 12y_2 + 2y_3}{720\lambda^6}$$

$$\begin{aligned}\frac{dy}{dx} = \frac{1}{\lambda}\Bigg[&\left\{ \frac{20}{3} - \frac{254}{9}\left(\frac{x}{\lambda}\right) + 31\left(\frac{x}{\lambda}\right)^2 - \frac{121}{9}\left(\frac{x}{\lambda}\right)^3 + \frac{5}{2}\left(\frac{x}{\lambda}\right)^4 - \frac{1}{6}\left(\frac{x}{\lambda}\right)^5 \right\} y_o \\ &+ \left\{ -\frac{135}{12} + \frac{1647}{36}\left(\frac{x}{\lambda}\right) - 48\left(\frac{x}{\lambda}\right)^2 + \frac{61}{3}\left(\frac{x}{\lambda}\right)^3 - \frac{15}{4}\left(\frac{x}{\lambda}\right)^4 + \frac{1}{4}\left(\frac{x}{\lambda}\right)^5 \right\} y_1 \\ &+ \left\{ \frac{36}{5} - \frac{342}{15}\left(\frac{x}{\lambda}\right) + 21\left(\frac{x}{\lambda}\right)^2 - \frac{25}{3}\left(\frac{x}{\lambda}\right)^3 + \frac{3}{2}\left(\frac{x}{\lambda}\right)^4 - \frac{1}{10}\left(\frac{x}{\lambda}\right)^5 \right\} y_2 \\ &+ \left\{ -\frac{157}{60} + \frac{949}{180}\left(\frac{x}{\lambda}\right) - 4\left(\frac{x}{\lambda}\right)^2 + \frac{13}{9}\left(\frac{x}{\lambda}\right)^3 - \frac{1}{4}\left(\frac{x}{\lambda}\right)^4 + \frac{1}{60}\left(\frac{x}{\lambda}\right)^5 \right\} y_3 \Bigg]\end{aligned}$$

$$\begin{aligned}\frac{d^2y}{dx^2} = \frac{1}{\lambda^2}\Bigg[&\left\{ -\frac{254}{9} + 62\left(\frac{x}{\lambda}\right) - \frac{121}{3}\left(\frac{x}{\lambda}\right)^2 + 10\left(\frac{x}{\lambda}\right)^3 - \frac{5}{6}\left(\frac{x}{\lambda}\right)^4 \right\} y_o \\ &+ \left\{ \frac{1647}{36} - 96\left(\frac{x}{\lambda}\right) + 61\left(\frac{x}{\lambda}\right)^2 - 15\left(\frac{x}{\lambda}\right)^3 + \frac{5}{4}\left(\frac{x}{\lambda}\right)^4 \right\} y_1 \\ &+ \left\{ -\frac{342}{15} + 42\left(\frac{x}{\lambda}\right) - 25\left(\frac{x}{\lambda}\right)^2 + 6\left(\frac{x}{\lambda}\right)^3 - \frac{1}{2}\left(\frac{x}{\lambda}\right)^4 \right\} y_2 \\ &+ \left\{ \frac{949}{180} - 8\left(\frac{x}{\lambda}\right) + \frac{13}{3}\left(\frac{x}{\lambda}\right)^2 - \left(\frac{x}{\lambda}\right)^3 + \frac{1}{12}\left(\frac{x}{\lambda}\right)^4 \right\} y_3 \Bigg]\end{aligned}$$

The equations on the previous page for the first and second derivative of a symmetrical sixth order polynomial are evaluated below at the seven values of x equal to o, λ, 2λ, 3λ, 4λ, 5λ, and 6λ.

$$x = o, \quad \frac{dy}{dx} = \left[\frac{20}{3}y_o - \frac{135}{12}y_1 + \frac{36}{5}y_2 - \frac{157}{60}y_3\right]/\lambda,$$

$$\frac{d^2y}{dx^2} = \frac{1}{\lambda^2}\left[-\frac{254}{9}y_o + \frac{1647}{36}y_1 - \frac{114}{5}y_2 + \frac{949}{180}y_3\right]$$

$$x = \lambda, \quad \frac{dy}{dx} = \left[-\frac{5}{3}y_o + \frac{10}{3}y_1 - \frac{23}{15}y_2 - \frac{2}{15}y_3\right]/\lambda,$$

$$\frac{d^2y}{dx^2} = \left[\frac{47}{18}y_o - 3y_1 - \frac{3}{10}y_2 + \frac{31}{45}y_3\right]/\lambda^2$$

$$x = 2\lambda, \quad \frac{dy}{dx} = \left[\frac{4}{3}y_o - \frac{13}{12}y_1 - \frac{4}{15}y_2 + \frac{1}{60}y_3\right]/\lambda,$$

$$\frac{d^2y}{dx^2} = \left[\frac{10}{9}y_o - \frac{9}{4}y_1 + \frac{6}{5}y_2 - \frac{11}{180}y_3\right]/\lambda^2$$

$$x = 3\lambda, \quad \frac{dy}{dx} = \left[0y_o + 0y_1 + 0y_2 + 0y_3\right]/\lambda,$$

$$\frac{d^2y}{dx^2} = \left[-\frac{49}{18}y_o + 3y_1 - \frac{3}{10}y_2 + \frac{1}{45}y_3\right]/\lambda^2$$

$$x = 4\lambda, \quad \frac{dy}{dx} = \left[-\frac{4}{3}y_o + \frac{13}{12}y_1 + \frac{4}{15}y_2 - \frac{1}{60}y_3\right]/\lambda,$$

$$\frac{d^2y}{dx^2} = \left[\frac{10}{9}y_o - \frac{9}{4}y_1 + \frac{6}{5}y_2 - \frac{11}{180}y_3\right]/\lambda^2$$

$$x = 5\lambda, \quad \frac{dy}{dx} = \left[\frac{5}{3}y_o - \frac{10}{3}y_1 + \frac{23}{15}y_2 + \frac{2}{15}y_3\right]/\lambda,$$

$$\frac{d^2y}{dx^2} = \left[\frac{47}{18}y_o - 3y_1 - \frac{3}{10}y_2 + \frac{31}{45}y_3\right]/\lambda^2$$

$$x = 6\lambda, \quad \frac{dy}{dx} = \left[-\frac{20}{3}y_o + \frac{135}{12}y_1 - \frac{36}{5}y_2 + \frac{157}{60}y_3 \right] /\lambda,$$

$$\frac{d^2y}{dx^2} = \left[-\frac{254}{9}y_o + \frac{1647}{36}y_1 - \frac{114}{5}y_2 + \frac{949}{180}y_3 \right] /\lambda^2$$

FINITE DIFFERENCE EQUATIONS FOR SIMPLY SUPPORTED, UNIFORMLY LOADED SQUARE PLATE

The finite difference equations mentioned on page 10 are shown below and on the following page. The finite difference equations at the six interior net points corresponding to the differential equation

$$\frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2} = -q$$

are

$$\begin{aligned}
4M_1 - 4M_o &= -qa^2/36 \\
2M_2 + M_3 + M_o - 4M_1 &= -qa^2/36 \\
2M_4 + 2M_1 - 4M_2 &= -qa^2/36 \\
2M_4 + M_1 - 4M_3 &= -qa^2/36 \\
M_2 + M_3 + M_5 - 4M_4 &= -qa^2/36 \\
2M_4 - 4M_5 &= -qa^2/36
\end{aligned}$$

The finite difference equations at the six interior net points corresponding to the differential equation

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = -\frac{M}{D}$$

are

$$4w_1 - 4w_o + M_o a^2/36D = 0$$

$$2w_2 + w_3 + w_o - 4w_1 + M_1 a^2/36D = 0$$

$$2w_4 + 2w_1 - 4w_2 + M_2 a^2/36D = 0$$

$$2w_4 + w_1 - 4w_3 + M_3 a^2/36D = 0$$

$$w_2 + w_3 + w_5 - 4w_4 + M_4 a^2/36D = 0$$

$$2w_4 - 4w_5 + M_5 a^2/36D = 0$$

Assigning the values a = 1, q = 36, D = 1/36, the following answers were arrived at with the use of a digital computer.

$M_o = 2.5961536,$	$w_o = 5.2466708$
$M_1 = 2.3461536,$	$w_1 = 4.5976324$
$M_2 = 2.1249998,$	$w_2 = 4.0312494$
$M_3 = 1.5384614,$	$w_3 = 2.7352066$
$M_4 = 1.4038461,$	$w_4 = 2.4023665$
$M_5 = .95192303,$	$w_5 = 1.4391640$

$$w_o = (5.2466708/36^2)\ \frac{qa^4}{D} = .004048\ qa^4/D$$

$$w_o = .004048\ \frac{12}{1-.3^2}\ \frac{Eh^3}{qa^4} = .044208\ \frac{Eh^3}{qa^4}$$

$$M_o = (2.5961536/36)\ qa^2 = .072115\ qa^2$$

$$Mx_o = My_o = M_o\ \frac{1 + \nu}{2} \qquad = .046875\ qa^2$$

POLYNOMIAL DIFFERENCE EQUATIONS FOR A SIMPLY SUPPORTED, UNFORMLY LOADED, SQUARE PLATE

The polynomial difference equations mentioned on page 11 are shown below and on the following page. These equations, corresponding to the differential equation

$$\frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2} = -q,$$

are[1]

$$-\frac{245}{90}M_o + \frac{270}{90}M_1 - \frac{27}{90}M_3 - \frac{245}{90}M_o + \frac{270}{90}M_1 - \frac{27}{90}M_3 = -qa^2/36$$

$$\frac{200}{180}M_o - \frac{405}{180}M_1 + \frac{216}{180}M_3 - \frac{245}{90}M_1 + \frac{270}{90}M_2 - \frac{27}{90}M_4 = -qa^2/36$$

$$\frac{200}{180}M_1 - \frac{405}{180}M_2 + \frac{216}{180}M_4 - \frac{200}{180}M_1 - \frac{405}{180}M_2 + \frac{216}{180}M_4 = -qa^2/36$$

$$\frac{235}{90}M_o - \frac{270}{90}M_1 - \frac{27}{90}M_3 - \frac{245}{90}M_3 + \frac{270}{90}M_4 - \frac{27}{90}M_5 = -qa^2/36$$

$$\frac{235}{90}M_1 - \frac{270}{90}M_2 - \frac{27}{90}M_4 + \frac{200}{180}M_3 - \frac{405}{180}M_4 + \frac{216}{180}M_5 = -qa^2/36$$

$$\frac{235}{90}M_3 - \frac{270}{90}M_4 - \frac{27}{90}M_5 + \frac{235}{90}M_3 - \frac{270}{90}M_4 - \frac{27}{90}M_5 = -qa^2/36$$

The polynomial difference equations corresponding to the differential equation

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = -\frac{M}{D}$$

---

[1]The first three terms of each equation represent the partial with respect to x, and the next three the partial with respectto y.

are

$$-\frac{245}{90}w_o + \frac{270}{90}w_1 - \frac{27}{90}w_3 - \frac{245}{90}w_o + \frac{270}{90}w_1 - \frac{27}{90}w_3 + M_o a^2/36D = 0$$

$$\frac{200}{180}w_o - \frac{405}{180}w_1 + \frac{216}{180}w_3 - \frac{245}{90}w_1 + \frac{270}{90}w_2 - \frac{27}{90}w_4 + M_1 a^2/36D = 0$$

$$\frac{200}{180}w_1 - \frac{405}{180}w_2 + \frac{216}{180}w_4 + \frac{200}{180}w_1 - \frac{405}{180}w_2 + \frac{216}{180}w_4 + M_2 a^2/36D = 0$$

$$\frac{235}{90}w_o - \frac{270}{90}w_1 - \frac{27}{90}w_3 - \frac{245}{90}w_3 + \frac{270}{90}w_4 - \frac{27}{90}w_5 + M_3 a^2/36D = 0$$

$$\frac{235}{90}w_1 - \frac{270}{90}w_2 - \frac{27}{90}w_4 + \frac{200}{180}w_3 - \frac{405}{180}w_4 + \frac{216}{180}w_5 + M_4 a^2/36D = 0$$

$$\frac{235}{90}w_3 - \frac{270}{90}w_4 - \frac{27}{90}w_5 + \frac{235}{90}w_3 - \frac{270}{90}w_4 - \frac{27}{90}w_5 + M_5 a^2/36D = 0$$

Assigning the values $a = 1$, $q = 36$, $D = 1/36$, the following answers were arrived at with the use of a digital computer.

$M_o = 2.6520986$	$w_o = 5.2665207$
$M_1 = 2.3970495$	$w_1 = 4.6100678$
$M_2 = 2.1722469$	$w_2 = 4.0381481$
$M_3 = 1.5718241$	$w_3 = 2.7320420$
$M_4 = 1.4368058$	$w_4 = 2.3978437$
$M_5 = .97930060$	$w_5 = 1.4326153$

$$w_o = 5.2665207 \frac{12}{36^2(1 - .3^2)} = .044375 \frac{Eh^3}{qa^4}$$

$$Mx_o = 2.6520986 \frac{1 + .3}{2 \times 36} = .047885\ qa^2$$

DERIVATION OF COEFFICIENTS OF A SYMMETRICAL TENTH ORDER POLYNOMIAL

The coefficients of a symmetrical tenth order polynomial are arrived at in a manner similar to that for the sixth order symmetrical polynomial. Letting y have the values y_5, y_4, y_3, y_2, y_1, y_o, y_1, y_2, y_3, y_4, y_5 at the values of x equal to o, λ, 2λ, 3λ, 4λ, 5λ, 6λ, 7λ, 8λ, 9λ, 10λ, the equation $y = f(x)$ is derived as follows:

$$A_1 = \frac{y_4 - y_5}{\lambda}, \quad A_2 = \frac{y_3 - 2y_4 + y_5}{2\lambda^2}, \quad A_3 = \frac{y_2 - 3y_3 + 3y_4 - y_5}{6\lambda^3}$$

$$A_4 = \frac{y_1 - 4y_2 + 6y_3 - 4y_4 + y_5}{24\lambda^4}, \quad A_5 = \frac{y_o - 5y_1 + 10y_2 - 10y_3 + 5y_4 - y_5}{120\lambda^5}$$

$$A_6 = \frac{-6y_o + 16y_1 - 20y_2 + 15y_3 - 6y_4 + y_5}{720\lambda^6}$$

$$A_7 = \frac{21y_o - 42y_1 + 36y_2 - 21y_3 + 7y_4 - y_5}{5,040\lambda^7}$$

$$A_8 = \frac{-56y_o + 98y_1 - 64y_2 + 29y_3 - 8y_4 + y_5}{40,320\lambda^8}$$

$$A_9 = \frac{126y_o - 210y_1 + 120y_2 - 45y_3 + 10y_4 - y_5}{362,880\lambda^9}$$

$$A_{10} = \frac{-252y_o + 420y_1 - 240y_2 + 90y_3 - 20y_4 + 2y_5}{3,628,800\lambda^{10}}$$

Again using the general formula on page 34, the equation on the following page can be arrived at after considerable arithmetic and algebraic manipulation.

$$
\begin{aligned}
y = \Bigg[& \frac{1}{14400}\Bigg\{725760\left(\frac{x}{\lambda}\right) - 1980576\left(\frac{x}{\lambda}\right)^2 + 2154600\left(\frac{x}{\lambda}\right)^3 - 1250980\left(\frac{x}{\lambda}\right)^4 + 433190\left(\frac{x}{\lambda}\right)^5 \\
& \quad - 93773\left(\frac{x}{\lambda}\right)^6 + 12800\left(\frac{x}{\lambda}\right)^7 - 1070\left(\frac{x}{\lambda}\right)^8 + 50\left(\frac{x}{\lambda}\right)^9 - \left(\frac{x}{\lambda}\right)^{10}\Bigg\} y_o \\
& + \frac{1}{3628800}\Bigg\{- 317520000\left(\frac{x}{\lambda}\right) + 861210000\left(\frac{x}{\lambda}\right)^2 - 929871600\left(\frac{x}{\lambda}\right)^3 + 535688580\left(\frac{x}{\lambda}\right)^4 \\
& \quad - 184199400\left(\frac{x}{\lambda}\right)^5 + 39655980\left(\frac{x}{\lambda}\right)^6 - 5392800\left(\frac{x}{\lambda}\right)^7 + 449820\left(\frac{x}{\lambda}\right)^8 \\
& \quad - 21000\left(\frac{x}{\lambda}\right)^9 + 420\left(\frac{x}{\lambda}\right)^{10}\Bigg\} y_1 \\
& + \frac{1}{3628800}\Bigg\{207360000\left(\frac{x}{\lambda}\right) - 550080000\left(\frac{x}{\lambda}\right)^2 + 578428800\left(\frac{x}{\lambda}\right)^3 - 324661440\left(\frac{x}{\lambda}\right)^4 \\
& \quad + 109216800\left(\frac{x}{\lambda}\right)^5 - 23128560\left(\frac{x}{\lambda}\right)^6 + 3110400\left(\frac{x}{\lambda}\right)^7 - 257760\left(\frac{x}{\lambda}\right)^8 \\
& \quad + 12000\left(\frac{x}{\lambda}\right)^9 - 240\left(\frac{x}{\lambda}\right)^{10}\Bigg\} y_2 \\
& + \frac{1}{3628800}\Bigg\{-102060000\left(\frac{x}{\lambda}\right) + 255555000\left(\frac{x}{\lambda}\right)^2 - 252379800\left(\frac{x}{\lambda}\right)^3 + 134546490\left(\frac{x}{\lambda}\right)^4 \\
& \quad - 43539300\left(\frac{x}{\lambda}\right)^5 + 8969310\left(\frac{x}{\lambda}\right)^6 - 1184400\left(\frac{x}{\lambda}\right)^7 + 97110\left(\frac{x}{\lambda}\right)^8 - 4500\left(\frac{x}{\lambda}\right)^9 \\
& \quad + 90\left(\frac{x}{\lambda}\right)^{10}\Bigg\} y_3 \\
& + \frac{1}{3628800}\Bigg\{40320000\left(\frac{x}{\lambda}\right) - 81360000\left(\frac{x}{\lambda}\right)^2 + 70445600\left(\frac{x}{\lambda}\right)^3 - 34467280\left(\frac{x}{\lambda}\right)^4 \\
& \quad + 10529400\left(\frac{x}{\lambda}\right)^5 - 2086980\left(\frac{x}{\lambda}\right)^6 + 268800\left(\frac{x}{\lambda}\right)^7 - 21720\left(\frac{x}{\lambda}\right)^8 + 1000\left(\frac{x}{\lambda}\right)^9 - 20\left(\frac{x}{\lambda}\right)^{10}\Bigg\} y_4
\end{aligned}
$$

$$+ \frac{1}{3628800}\left\{3628800-10991520\left(\frac{x}{\lambda}\right)+13780152\left(\frac{x}{\lambda}\right)^2-9582200\left(\frac{x}{\lambda}\right)^3+4140610\left(\frac{x}{\lambda}\right)^4\right.$$

$$\left.-1171380\left(\frac{x}{\lambda}\right)^5+221046\left(\frac{x}{\lambda}\right)^6-27600\left(\frac{x}{\lambda}\right)^7+2190\left(\frac{x}{\lambda}\right)^8-100\left(\frac{x}{\lambda}\right)^9+2\left(\frac{x}{\lambda}\right)^{10}\right\} y_5 \Bigg]$$

$$\frac{dy}{dx} = \frac{1}{\lambda}\Bigg[\frac{1}{14400}\left\{725760-3961152\left(\frac{x}{\lambda}\right)+6463800\left(\frac{x}{\lambda}\right)^2-5003920\left(\frac{x}{\lambda}\right)^3+2165950\left(\frac{x}{\lambda}\right)^4\right.$$

$$\left.-562638\left(\frac{x}{\lambda}\right)^5+89600\left(\frac{x}{\lambda}\right)^6-8560\left(\frac{x}{\lambda}\right)^7+450\left(\frac{x}{\lambda}\right)^8-10\left(\frac{x}{\lambda}\right)^9\right\} y_0$$

$$+ \frac{1}{3628800}\left\{-317520000+1722420000\left(\frac{x}{\lambda}\right)-2789614800\left(\frac{x}{\lambda}\right)^2+2142754320\left(\frac{x}{\lambda}\right)^3\right.$$

$$-920997000\left(\frac{x}{\lambda}\right)^4+237935880\left(\frac{x}{\lambda}\right)^5-37749600\left(\frac{x}{\lambda}\right)^6+3598560\left(\frac{x}{\lambda}\right)^7-189000\left(\frac{x}{\lambda}\right)^8$$

$$\left.+4200\left(\frac{x}{\lambda}\right)^9\right\} y_1$$

$$+ \frac{1}{3628800}\left\{207360000-1100160000\left(\frac{x}{\lambda}\right)+1735286400\left(\frac{x}{\lambda}\right)^2-1298645760\left(\frac{x}{\lambda}\right)^3\right.$$

$$+546084000\left(\frac{x}{\lambda}\right)^4-138771360\left(\frac{x}{\lambda}\right)^5+21772800\left(\frac{x}{\lambda}\right)^6-2062080\left(\frac{x}{\lambda}\right)^7$$

$$\left.+108000\left(\frac{x}{\lambda}\right)^8-2400\left(\frac{x}{\lambda}\right)^9\right\} y_2$$

$$+ \frac{1}{3628800}\left\{-102060000 + 511110000\left(\frac{x}{\lambda}\right)-757139400\left(\frac{x}{\lambda}\right)^2+538185960\left(\frac{x}{\lambda}\right)^3\right.$$

$$-217696500\left(\frac{x}{\lambda}\right)^4+53815860\left(\frac{x}{\lambda}\right)^5-8290800\left(\frac{x}{\lambda}\right)^6+776880\left(\frac{x}{\lambda}\right)^7-40500\left(\frac{x}{\lambda}\right)^8$$

$$\left.+900\left(\frac{x}{\lambda}\right)^9\right\} y_3$$

$$+ \frac{1}{3628800}\left\{40320000-162720000\left(\frac{x}{\lambda}\right)+211336800\left(\frac{x}{\lambda}\right)^2-137869120\left(\frac{x}{\lambda}\right)^3\right.$$

$$\left.+52647000\left(\frac{x}{\lambda}\right)^4-12521880\left(\frac{x}{\lambda}\right)^5+1881600\left(\frac{x}{\lambda}\right)^6-173760\left(\frac{x}{\lambda}\right)^7+9000\left(\frac{x}{\lambda}\right)^8-200\left(\frac{x}{\lambda}\right)^9\right\} y_4$$

$$+\frac{1}{3628800}\left\{-10991520+27560304\left(\frac{x}{\lambda}\right)-28746600\left(\frac{x}{\lambda}\right)^{2}+16562440\left(\frac{x}{\lambda}\right)^{3}-5856900\left(\frac{x}{\lambda}\right)^{4}\right.$$

$$\left.+1326276\left(\frac{x}{\lambda}\right)^{5}-193200\left(\frac{x}{\lambda}\right)^{6}+17520\left(\frac{x}{\lambda}\right)^{7}-900\left(\frac{x}{\lambda}\right)^{8}+20\left(\frac{x}{\lambda}\right)^{9}\right\}y_5\Bigg]$$

$$\frac{d^2y}{dx^2}=\frac{1}{\lambda^2}\Bigg[\frac{1}{14400}\left\{-3961152+12927600\left(\frac{x}{\lambda}\right)-15011760\left(\frac{x}{\lambda}\right)^{2}+8663800\left(\frac{x}{\lambda}\right)^{3}\right.$$

$$\left.-2813190\left(\frac{x}{\lambda}\right)^{4}+537600\left(\frac{x}{\lambda}\right)^{5}-59920\left(\frac{x}{\lambda}\right)^{6}+3600\left(\frac{x}{\lambda}\right)^{7}-90\left(\frac{x}{\lambda}\right)^{8}\right\}y_o$$

$$+\frac{1}{3628800}\left\{1722420000-5579229600\left(\frac{x}{\lambda}\right)+6428262960\left(\frac{x}{\lambda}\right)^{2}-3683988000\left(\frac{x}{\lambda}\right)^{3}\right.$$

$$\left.+1189679400\left(\frac{x}{\lambda}\right)^{4}-226497600\left(\frac{x}{\lambda}\right)^{5}+25189920\left(\frac{x}{\lambda}\right)^{6}-1512000\left(\frac{x}{\lambda}\right)^{7}+37800\left(\frac{x}{\lambda}\right)^{8}\right\}y_1$$

$$+\frac{1}{3628800}\left\{-1100160000+3470512800\left(\frac{x}{\lambda}\right)-3895937280\left(\frac{x}{\lambda}\right)^{2}+2184336000\left(\frac{x}{\lambda}\right)^{3}\right.$$

$$\left.-693856800\left(\frac{x}{\lambda}\right)^{4}+130636800\left(\frac{x}{\lambda}\right)^{5}-14434560\left(\frac{x}{\lambda}\right)^{6}+864000\left(\frac{x}{\lambda}\right)^{7}-21600\left(\frac{x}{\lambda}\right)^{8}\right\}y_2$$

$$+\frac{1}{3628800}\left\{511110000-1514278800\left(\frac{x}{\lambda}\right)+1614557880\left(\frac{x}{\lambda}\right)^{2}-870786000\left(\frac{x}{\lambda}\right)^{3}\right.$$

$$\left.+269079300\left(\frac{x}{\lambda}\right)^{4}-49744800\left(\frac{x}{\lambda}\right)^{5}+5438160\left(\frac{x}{\lambda}\right)^{6}-324000\left(\frac{x}{\lambda}\right)^{7}+8100\left(\frac{x}{\lambda}\right)^{8}\right\}y_3$$

$$+\frac{1}{3628800}\left\{-162720000+422673600\left(\frac{x}{\lambda}\right)-413607360\left(\frac{x}{\lambda}\right)^{2}+210588000\left(\frac{x}{\lambda}\right)^{3}\right.$$

$$\left.-62609400\left(\frac{x}{\lambda}\right)^{4}+11289600\left(\frac{x}{\lambda}\right)^{5}-1216320\left(\frac{x}{\lambda}\right)^{6}+72000\left(\frac{x}{\lambda}\right)^{7}-1800\left(\frac{x}{\lambda}\right)^{8}\right\}y_4$$

$$+\frac{1}{3628800}\left\{27560304-57493200\left(\frac{x}{\lambda}\right)+49687320\left(\frac{x}{\lambda}\right)^{2}-23427600\left(\frac{x}{\lambda}\right)^{3}+6631380\left(\frac{x}{\lambda}\right)^{4}\right.$$

$$\left.-1159200\left(\frac{x}{\lambda}\right)^{5}+122640\left(\frac{x}{\lambda}\right)^{6}-7200\left(\frac{x}{\lambda}\right)^{7}+180\left(\frac{x}{\lambda}\right)^{8}\right\}y_5\Bigg]$$

The equations on the previous two pages for the first and second derivative of a symmetrical tenth order polynomial are evaluated below and on the next page for the eleven values of x from zero through 10λ.

$$x = 0,\ \frac{dy}{dx} = \left[50\frac{2}{5}y_0 - 87\frac{1}{2}y_1 + 57\frac{1}{7}y_2 - 28\frac{1}{8}y_3 + 11\frac{1}{9}y_4 - 3\ \frac{73}{2520}y_5\right]\frac{1}{\lambda}$$

$$x = \lambda,\ \frac{dy}{dx} = \left[-6\frac{3}{10}y_0 + 11\frac{1}{5}y_1 - 8y_2 + 5\frac{1}{7}y_3 - 1\frac{601}{630}y_4 - \frac{4}{45}y_5\right]\frac{1}{\lambda}$$

$$x = 2\lambda,\ \frac{dy}{dx} = \left[1\frac{13}{15}y_0 - 3\frac{1}{2}y_1 + 3\frac{1}{5}y_2 - 1\frac{323}{840}y_3 - \frac{4}{21}y_4 + \frac{1}{120}y_5\right]\frac{1}{\lambda}$$

$$x = 3\lambda,\ \frac{dy}{dx} = \left[-1\frac{1}{20}y_0 + 2\frac{1}{3}y_1 - 1\frac{1}{105}y_2 - \frac{3}{10}y_3 + \frac{7}{252}y_4 - \frac{1}{630}y_5\right]\frac{1}{\lambda}$$

$$x = 4\lambda,\ \frac{dy}{dx} = \left[1\frac{1}{5}y_0 - \frac{91}{105}y_1 - \frac{8}{21}y_2 + \frac{3}{56}y_3 - \frac{2}{315}y_4 + \frac{1}{2520}y_5\right]\frac{1}{\lambda}$$

$$x = 5\lambda,\ \frac{dy}{dx} = \left[0y_0 + 0y_1 + 0y_2 + 0y_3 + 0y_4 + 0y_5\right]\frac{1}{\lambda}$$

$$x = 6\lambda,\ \frac{dy}{dx} = \left[-1\frac{1}{5}y_0 + \frac{91}{105}y_1 + \frac{8}{21}y_2 - \frac{3}{56}y_3 + \frac{2}{315}y_4 - \frac{1}{2520}y_5\right]\frac{1}{\lambda}$$

$$x = 7\lambda,\ \frac{dy}{dx} = \left[1\frac{1}{20}y_0 - 2\frac{1}{3}y_1 + 1\frac{1}{105}y_2 + \frac{3}{10}y_3 - \frac{7}{252}y_4 + \frac{1}{630}y_5\right]\frac{1}{\lambda}$$

$$x = 8\lambda,\ \frac{dy}{dx} = \left[-1\frac{13}{15}y_0 + 3\frac{1}{2}y_1 - 3\frac{1}{5}y_2 + 1\frac{323}{840}y_3 + \frac{4}{21}y_4 - \frac{1}{120}y_5\right]\frac{1}{\lambda}$$

$$x = 9\lambda,\ \frac{dy}{dx} = \left[6\frac{3}{10}y_0 - 11\frac{1}{5}y_1 + 8y_2 - 5\frac{1}{7}y_3 + 1\frac{601}{630}y_4 + \frac{4}{45}y_5\right]\frac{1}{\lambda}$$

$$x = 10\lambda,\ \frac{dy}{dx} = \left[-50\frac{2}{5}y_0 + 87\frac{1}{2}y_1 - 57\frac{1}{7}y_2 + 28\frac{1}{8}y_3 - 11\frac{1}{9}y_4 + 3\frac{73}{2520}y_5\right]\frac{1}{\lambda}$$

$$x = 0,\ \frac{d^2y}{dx^2} = \left[-275\frac{2}{25}y_0 + 474\frac{47}{72}y_1 - 303\frac{11}{63}y_2 + 140\frac{95}{112}y_3 - 44\frac{53}{63}y_4 + 7\frac{4997}{8400}y_5\right]\frac{1}{\lambda^2}$$

$$x = \lambda,\ \frac{d^2y}{dx^2} = \left[19\,\frac{179}{200}y_o - 34\,\frac{28}{45}y_1 + 22\,\frac{188}{315}y_2 - 9\,\frac{22}{35}y_3 + 1\,\frac{583}{2520}y_4 + \frac{277}{525}y_5\right]\frac{1}{\lambda^2}$$

$$x = 2\lambda,\ \frac{d^2y}{dx^2} = \left[-3\,\frac{68}{225}y_o + 5\,\frac{73}{120}y_1 - 2\,\frac{26}{105}y_2 - \frac{4757}{5040}y_3 + \frac{289}{315}y_4 - \frac{809}{25200}y_5\right]\frac{1}{\lambda^2}$$

$$x = 3\lambda,\ \frac{d^2y}{dx^2} = \left[\frac{109}{200}y_o + \frac{31}{90}y_1 - 2\,\frac{13}{315}y_2 + 1\,\frac{33}{140}y_3 - \frac{223}{2520}y_4 + \frac{29}{6300}y_5\right]\frac{1}{\lambda^2}$$

$$x = 4\lambda,\ \frac{d^2y}{dx^2} = \left[1\,\frac{13}{25}y_o - 2\,\frac{329}{360}y_1 + 1\,\frac{173}{315}y_2 - \frac{97}{560}y_3 + \frac{2}{105}y_4 - \frac{29}{25200}y_5\right]\frac{1}{\lambda^2}$$

$$x = 5\lambda,\ \frac{d^2y}{dx^2} = \left[-2\,\frac{1669}{1800}y_o + 3\,\frac{1}{3}y_1 - \frac{10}{21}y_2 + \frac{5}{63}y_3 - \frac{5}{504}y_4 + \frac{1}{1575}y_5\right]\frac{1}{\lambda^2}$$

$$x = 6\lambda,\ \frac{d^2y}{dx^2} = \left[1\,\frac{13}{25}y_o - 2\,\frac{329}{360}y_1 + 1\,\frac{173}{315}y_2 - \frac{97}{560}y_3 + \frac{2}{105}y_4 - \frac{29}{25200}y_5\right]\frac{1}{\lambda^2}$$

$$x = 7\lambda,\ \frac{d^2y}{dx^2} = \left[\frac{109}{200}y_o + \frac{31}{90}y_1 - 2\,\frac{13}{315}y_2 + 1\,\frac{33}{140}y_3 - \frac{223}{2520}y_4 + \frac{29}{6300}y_5\right]\frac{1}{\lambda^2}$$

$$x = 8\lambda,\ \frac{d^2y}{dx^2} = \left[-3\,\frac{68}{225}y_o + 5\,\frac{73}{120}y_1 - 2\,\frac{26}{105}y_2 - \frac{4757}{5040}y_3 + \frac{289}{315}y_4 - \frac{809}{25200}y_5\right]\frac{1}{\lambda^2}$$

$$x = 9\lambda,\ \frac{d^2y}{dx^2} = \left[19\,\frac{179}{200}y_o - 34\,\frac{28}{45}y_1 + 22\,\frac{188}{315}y_2 - 9\,\frac{22}{35}y_3 + 1\,\frac{583}{2520}y_4 + \frac{277}{525}y_5\right]\frac{1}{\lambda^2}$$

$$x = 10\lambda,\ \frac{d^2y}{dx^2} = \left[-275\,\frac{2}{25}y_o + 474\,\frac{47}{72}y_1 - 303\,\frac{11}{63}y_2 + 140\,\frac{95}{112}y_3 - 44\,\frac{53}{63}y_4 + 7\,\frac{4997}{8400}y_5\right]\frac{1}{\lambda^2}$$

POLYNOMIAL DIFFERENCE EQUATIONS

FOR CHAPTER IV CANTILEVER PLATE PROBLEM

The equations following are those indicated on page 13, and are derived using the coefficients from Tables 5 and 6. The same constants as mentioned in Chapter II are substituted for the parameters in the equations to permit solution by digital computer, namely

$a = 2,\ \lambda = a/2 = 1,\ \ N = [Eh^3/12(1-\nu^2)] = 10^3,\ \ P = 250,\ \nu = 0.30$

Holl
ref.
eq.

1^{st} of [1] $$\frac{200}{180}U_5 - \frac{405}{180}U_1 + \frac{216}{180}U_a - \frac{11}{180}U_\alpha$$

$$-\frac{83216}{25200}U_4 + \frac{141330}{25200}U_3 - \frac{56640}{25200}U_2 - \frac{23875}{25200}U_1 + \frac{23120}{25200}U_B - \frac{809}{25200}U_F = 0$$

$$-80485U_1 - 56640U_2 + 141330U_3 - 83216U_4 + 28000U_5 + 30240U_a$$
$$+ 23120U_B - 809U_F - 1540U_\alpha = 0$$

2^{nd} of [1] $$\frac{200}{180}U_6 - \frac{405}{180}U_2 + \frac{216}{180}U_b - \frac{11}{180}U_\beta$$

$$+\frac{6867}{12600}U_4 + \frac{4340}{12600}U_3 - \frac{25720}{12600}U_2 + \frac{15570}{12600}U_1 - \frac{1115}{12600}U_B + \frac{58}{12600}U_F = 0$$

$$15570U_1 - 54070U_2 + 4340U_3 + 6867U_4 + 14000U_6 + 15120U_a$$
$$-1115U_B + 58U_F - 770U_\beta = 0$$

3^{rd} of [1] $$\frac{200}{180}U_7 - \frac{405}{180}U_3 + \frac{216}{180}U_c - \frac{11}{180}U_\gamma$$

$$+\frac{38304}{25200}U_4 - \frac{73430}{25200}U_3 + \frac{39040}{25200}U_2 - \frac{4365}{25200}U_1 + \frac{480}{25200}U_B - \frac{29}{25200}U_F = 0$$

3rd of [1] $-4365U_1 + 39040U_2 - 130130U_3 + 383040U_4 + 28000U_7 + 30240U_c$

$$+ 480U_B - 29U_F - 1540U_\gamma = 0$$

4th of [1] $\frac{200}{180}U_8 - \frac{405}{180}U_4 + \frac{216}{180}U_d - \frac{11}{180}U_\delta$

$$- \frac{36883}{12600}U_4 + \frac{42000}{12600}U_3 - \frac{6000}{12600}U_2 + \frac{1000}{12600}U_1 - \frac{125}{12600}U_B + \frac{8}{12600}U_F = 0$$

$$1000U_1 - 6000U_2 + 42000U_3 - 65233U_4 + 14000U_8 + 15120U_d$$

$$- 125U_B + 8U_F - 770U_\delta = 0$$

1st of [2] $\frac{235}{90}U_5 - \frac{270}{90}U_1 - \frac{27}{90}U_a + \frac{62}{90}U_\alpha$

$$- \frac{83216}{25200}U_d + \frac{141330}{25200}U_c - \frac{56640}{25200}U_b - \frac{23785}{25200}U_a + \frac{23120}{25200}U_C - \frac{809}{25200}U_G = 0$$

$$- 75600U_1 + 65800U_5 - 31345U_a - 56640U_b + 141330U_a - 83216U_d$$

$$+ 23120U_C - 809U_G + 17360U_\alpha = 0$$

2nd of [2] $\frac{235}{90}U_6 - \frac{270}{90}U_2 - \frac{27}{90}U_b + \frac{62}{90}U_\beta$

$$+ \frac{6867}{12600}U_d + \frac{4340}{12600}U_c - \frac{25720}{12600}U_b + \frac{15570}{12600}U_a - \frac{1115}{12600}U_C + \frac{58}{12600}U_G = 0$$

$$- 37800U_2 + 32900U_6 + 15570U_a - 29500U_b + 4340U_c + 6867U_d$$

$$-1115U_C + 58U_G + 8680U_\beta = 0$$

3^{rd} of [2] $\frac{235}{90}U_7 - \frac{270}{90}U_3 - \frac{27}{90}U_c + \frac{62}{90}U_\gamma$

$$+ \frac{38304}{25200}U_d - \frac{73430}{25200}U_c + \frac{39040}{25200}U_b - \frac{4365}{25200}U_a + \frac{480}{25200}U_C - \frac{29}{25200}U_G = 0$$

$$- 75600U_3 + 65800U_7 - 4365U_a + 39040U_b - 80990U_c + 38304U_d$$

$$+ 480U_C - 29U_G + 17360U_\gamma = 0$$

4^{th} of [2] $\frac{235}{90}U_8 - \frac{270}{90}U_4 - \frac{27}{90}U_d + \frac{62}{90}U_\delta$

$$- \frac{36883}{12600}U_d + \frac{42000}{12600}U_c - \frac{6000}{12600}U_b + \frac{1000}{12600}U_a - \frac{125}{12600}U_C + \frac{8}{12600}U_G = 0$$

$$- 37800U_4 + 32900U_8 + 1000U_a - 6000U_b + 42000U_c - 40663U_d$$

$$- 125U_C + 8U_G + 8680U_\delta = 0$$

5^{th} of [2] $\frac{200}{180}U_A - \frac{405}{180}U_B + \frac{216}{180}U_C - \frac{11}{180}U_D$

$$+ \frac{250677}{12600}U_4 - \frac{436240}{12600}U_3 + \frac{284720}{12600}U_2 - \frac{121320}{12600}U_1 + \frac{15515}{12600}U_B + \frac{6648}{12600}U_F = 0$$

$$- 121320U_1 + 284720U_2 - 436240U_3 + 250677U_4 + 14000U_A - 12835U_B$$

$$+ 15120U_C - 770U_D + 6648U_F = 0$$

6^{th} of [2] $\frac{235}{90}U_A - \frac{270}{90}U_B - \frac{27}{90}U_C + \frac{62}{90}U_D$

$$+ \frac{250677}{12600}U_d - \frac{436240}{12600}U_c + \frac{284720}{12600}U_b - \frac{121320}{12600}U_a + \frac{15515}{12600}U_C + \frac{6648}{12600}U_G = 0$$

$$- 121320U_a + 284720U_b - 436240U_c + 250677U_d + 32900U_A - 37800U_B$$

$$+ 11735U_C + 8680U_D + 6648U_G = 0$$

1^{st} of [3] $25.2U_1 - 80485w_1 - 56640w_2 + 141330w_3 - 83216w_4 + 28000w_5$

$+ 30240w_a + 23120w_B - 809w_F - 1540w_\alpha = 0$

2^{nd} of [3] $12.6U_2 + 15570w_1 - 54070w_2 + 4340w_3 + 6867w_4 + 14000w_6$

$+ 15120w_b - 1115w_B + 58w_F - 770w_\beta = 0$

3^{rd} of [3] $25.2U_3 - 4365w_1 + 39040w_2 - 130130w_3 + 38304w_4 + 28000w_7$

$+ 30240w_c + 480w_B - 29w_F - 1540w_\gamma = 0$

4^{th} of [3] $12.6U_4 + 1000w_1 - 6000w_2 + 42000w_3 - 65233w_4 + 14000w_8$

$+ 15120w_d - 125w_B + 8w_F - 770w_\delta = 0$

1^{st} of [4] $25.2U_a - 75600w_1 + 65800w_5 - 31345w_a - 56640w_b + 141330w_c$

$- 83216w_d + 23120w_C - 809w_G + 17360w_\alpha = 0$

2^{nd} of [4] $12.6U_b - 37800w_2 + 32900w_6 + 15570w_a - 29500w_b + 4340w_c$

$+ 6867w_d - 1115w_C + 58w_G + 8680w_\beta = 0$

3^{rd} of [4] $25.2U_c - 75600w_3 + 65800w_7 - 4365w_a + 39040w_b - 80990w_c$

$+ 38304w_d + 480w_C - 29w_G + 17360w_\gamma = 0$

4^{th} of [4] $12.6U_d - 37800w_4 + 32900w_8 + 1000w_a - 6000w_b + 42000w_c$

$- 40663w_d - 125w_C + 8w_G + 8680w_\delta = 0$

5^{th} of [4] $12.6U_B - 121320w_1 + 284720w_2 - 436240w_3 + 250677w_4 + 14000w_A$

$- 12835w_B + 15120w_C - 770w_D + 6648w_F = 0$

6^{th} of [4] $12.6U_C - 121320w_a + 284720w_b - 436240w_c + 250677w_d + 32900w_A$

$-37800w_B + 11735w_C + 8680w_D + 6648w_G = 0$

1st of [6] $$.001U_5 - \frac{245}{90}w_5 + \frac{270}{90}w_1 - \frac{27}{90}w_a + \frac{2}{90}w_\alpha$$

$$- \frac{83216}{25200}w_8 + \frac{141330}{25200}w_7 - \frac{56640}{25200}w_6 - \frac{23785}{25200}w_5 + \frac{23120}{25200}w_A - \frac{809}{25200}w_E = 0$$

$$25.2U_5 + 75600w_1 - 92385w_5 - 56640w_6 + 141330w_7 - 83216w_8$$

$$- 7560w_a + 23120w_A - 809w_E + 560w_\alpha = 0$$

2nd of [6] $$.001U_6 - \frac{245}{90}w_6 + \frac{270}{90}w_2 - \frac{27}{90}w_b + \frac{2}{90}w_\beta$$

$$+ \frac{6867}{12600}w_8 + \frac{4340}{12600}w_7 - \frac{25720}{12600}w_6 + \frac{15570}{12600}w_5 - \frac{1115}{12600}w_A + \frac{58}{12600}w_E = 0$$

$$12.6U_6 + 37800w_2 + 15570w_5 - 60020w_6 + 4340w_7 + 6867w_8 - 3780w_b$$

$$-1115w_A + 58w_E + 280w_\beta = 0$$

3rd of [6] $$.001U_7 - \frac{245}{90}w_7 + \frac{270}{90}w_3 - \frac{27}{90}w_c + \frac{2}{90}w_\gamma$$

$$+ \frac{38304}{25200}w_8 - \frac{73430}{25200}w_7 + \frac{39040}{25200}w_6 - \frac{4365}{25200}w_5 + \frac{480}{25200}w_A - \frac{29}{25200}w_E = 0$$

$$25.2U_7 + 75600w_3 - 4365w_5 + 39040w_6 - 142030w_7 + 38304w_8$$

$$- 7560w_c + 480w_A - 29w_E + 560w_\gamma = 0$$

4th of [6] $$.001U_8 - \frac{245}{90}w_8 + \frac{270}{90}w_4 - \frac{27}{90}w_d + \frac{2}{90}w_\delta$$

$$- \frac{36883}{12600}w_8 + \frac{42000}{12600}w_7 - \frac{6000}{12600}w_6 + \frac{1000}{12600}w_5 - \frac{125}{12600}w_A + \frac{8}{12600}w_E = 0$$

$$12.6U_8 + 37800w_4 + 1000w_5 - 6000w_6 + 42000w_7 - 71183w_8$$

$$- 3780w_d - 125w_A + 8w_E + 280w_\delta = 0$$

5^{th} of [6] $.001U_A - \frac{245}{90}w_A + \frac{270}{90}w_B - \frac{27}{90}w_C + \frac{2}{90}w_D$

$+ \frac{250677}{12600}w_8 - \frac{436240}{12600}w_7 + \frac{284720}{12600}w_6 - \frac{121320}{12600}w_5 + \frac{15515}{12600}w_A + \frac{6648}{12600}w_E = 0$

$12.6U_A - 121320w_5 + 284720w_6 - 436240w_7 + 250677w_8 - 18785w_A$

$+ 37800w_B - 3780w_C + 280w_D + 6648w_E = 0$

1^{st} of [7] $.001U_a + .7\left[- \frac{83216}{25200}w_d + \frac{141330}{25200}w_c - \frac{56640}{25200}w_b - \frac{23785}{25200}w_a + \frac{23120}{25200}w_C \right.$

$\left. - \frac{809}{25200}w_G \right] = 0$

$25.2U_a - 58251.2w_d + 98931.w_c - 39648.w_b - 16649.5w_a + 16184.w_C$

$- 566.3w_G = 0$

2^{nd} of [7] $.001U_b + .7\left[\frac{6867}{12600}w_d + \frac{4340}{12600}w_c - \frac{25720}{12600}w_b + \frac{15570}{12600}w_a - \frac{1115}{12600}w_C \right.$

$\left. + \frac{58}{12600}w_G \right] = 0$

$12.6U_b + 4806.9w_d + 3038.w_c - 18004.w_b + 10899.w_a - 780.5w_C$

$+ 40.6w_G = 0$

3^{rd} of [7] $.001U_c + .7\left[\frac{38304}{25200}w_d - \frac{73430}{25200}w_c + \frac{39040}{25200}w_b - \frac{4365}{25200}w_a + \frac{480}{25200}w_C \right.$

$\left. - \frac{29}{25200}w_G \right] = 0$

$25.2U_c + 26812.8w_d - 51401.w_c + 27328.w_b - 3055.5w_a + 336.w_C$

$- 20.3w_G = 0$

4^{th} of [7] $.001U_d + .7\left[-\frac{36883}{12600}w_d + \frac{42000}{12600}w_c - \frac{6000}{12600}w_b + \frac{1000}{12600}w_a - \frac{125}{12600}w_C + \frac{8}{12600}w_G\right] = 0$

$$12.6U_d - 25818.1w_d + 29400.w_c - 4200.w_b + 700.w_a - 87.5w_C + 5.6w_G = 0$$

5^{th} of [7] $.001U_B + .7\left[\frac{200}{180}w_A - \frac{405}{180}w_B + \frac{216}{180}w_C - \frac{11}{180}w_D\right] = 0$

$$.18U_B + 140.w_A - 283.5w_B + 151.2w_C - 7.7w_D = 0$$

6^{th} of [7] $.001U_C + .7\left[\frac{250677}{12600}w_d - \frac{436240}{12600}w_c + \frac{284720}{12600}w_b - \frac{121320}{12600}w_a + \frac{15515}{12600}w_C + \frac{6648}{12600}w_G\right] = 0$

$$12.6U_C + 175473.9w_d - 305368.w_c + 199304.w_b - 84924.w_a + 10860.5w_C + 4653.6w_G = 0$$

7^{th} of [7] $.001U_C + .7\left[\frac{235}{90}w_A - \frac{270}{90}w_B - \frac{27}{90}w_C + \frac{62}{90}w_D\right] = 0$

$$.09U_C + 164.5w_A - 189.w_B - 18.9w_C + 43.4w_D = 0$$

The equation following is the fortieth equation added as mentioned on page 14. The equation is the polynomial difference equation setting M_y equal to zero at point A. The differential relationship is, as shown in general in Holl's (1) article and on page 22,

$$N(1-\nu)\left(\frac{\partial^2 w}{\partial x^2}\right)_A = -U_A$$

and the polynomial difference equation corresponding to this is

$$.001U_A + .7\left[-\frac{245}{90}w_A + \frac{270}{90}w_B - \frac{27}{90}w_C + \frac{2}{90}w_D\right]$$

$$= .09U_A - 171.5w_A + 189.w_B - 18.9w_C + 1.4w_D = 0$$

First of group [8]

$$\frac{25}{15}U_8 - \frac{50}{15}U_4 + \frac{23}{15}U_d + \frac{2}{15}U_\delta$$

$$\begin{aligned} - .7(1000)\Bigg[&\frac{25}{15}\left(-\frac{36883}{12600}w_8 + \frac{42000}{12600}w_7 - \frac{6000}{12600}w_6 + \frac{1000}{12600}w_5 - \frac{125}{12600}w_A + \frac{8}{12600}w_E\right) \\ &- \frac{50}{15}\left(-\frac{36883}{12600}w_4 + \frac{42000}{12600}w_3 - \frac{6000}{12600}w_2 + \frac{1000}{12600}w_1 - \frac{125}{12600}w_B + \frac{8}{12600}w_F\right) \\ &+ \frac{23}{15}\left(-\frac{36883}{12600}w_d + \frac{42000}{12600}w_c - \frac{6000}{12600}w_b + \frac{1000}{12600}w_a - \frac{125}{12600}w_C + \frac{8}{12600}w_G\right) \\ &+ \frac{2}{15}\left(-\frac{36883}{12600}w_\delta + \frac{42000}{12600}w_\gamma - \frac{6000}{12600}w_\beta + \frac{1000}{12600}w_\alpha - \frac{125}{12600}w_D + \frac{8}{12600}w_H\right)\Bigg] = 250 \end{aligned}$$

$$450U_8 - 900U_4 + 414U_d + 36U_\delta$$

$$+ 922075w_8 - 1050000w_7 + 150000w_6 - 25000w_5 + 3125w_A - 200w_E$$

$$- 1844150w_4 + 2100000w_3 - 300000w_2 + 50000w_1 - 6250w_B + 400w_F$$

$$+ 848309w_d - 966000w_c + 138000w_b - 23000w_a + 2875w_C - 184w_G$$

$$+ 73766w_\delta - 84000w_\gamma + 12000w_\beta - 2000w_\alpha + 250w_D - 16w_H = 67500$$

Second of group [8]

$$\begin{aligned}
&\frac{25}{15}U_7 - \frac{50}{15}U_3 + \frac{23}{15}U_c + \frac{2}{15}U_\gamma \\
&- .7(1000)\Bigg[\frac{25}{15}\left(\frac{38304}{25200}w_8 - \frac{73430}{25200}w_7 + \frac{39040}{25200}w_6 - \frac{4365}{25200}w_5 + \frac{480}{25200}w_A - \frac{29}{25200}w_E\right) \\
&\qquad - \frac{50}{15}\left(\frac{38304}{25200}w_4 - \frac{73430}{25200}w_3 + \frac{39040}{25200}w_2 - \frac{4365}{25200}w_1 + \frac{480}{25200}w_B - \frac{29}{25200}w_F\right) \\
&\qquad + \frac{23}{15}\left(\frac{38304}{25200}w_d - \frac{73430}{25200}w_c + \frac{39040}{25200}w_b - \frac{4365}{25200}w_a + \frac{480}{25200}w_C - \frac{29}{25200}w_G\right) \\
&\qquad + \frac{2}{15}\left(\frac{38304}{25200}w_\delta - \frac{73430}{25200}w_\gamma + \frac{39040}{25200}w_\beta - \frac{4365}{25200}w_\alpha + \frac{480}{25200}w_D - \frac{29}{25200}w_H\right)\Bigg] = 0
\end{aligned}$$

$$900U_7 - 1800U_3 + 828U_c + 72U_\gamma$$

$$- 957600w_8 + 1835750w_7 - 976000w_6 + 109125w_5 - 12000w_A + 725w_E$$

$$+ 1915200w_4 - 3671500w_3 + 1952000w_2 - 218250w_1 + 24000w_B - 1450w_F$$

$$- 880992w_d + 1688890w_c - 897920w_b + 100395w_a - 11040w_C + 667w_G$$

$$+ 76608w_\delta + 146860w_\gamma - 78080w_\beta + 8730w_\alpha - 960w_D + 58w_H = 0$$

Third of group [8]

$$\frac{25}{15}U_6 - \frac{50}{15}U_2 + \frac{23}{15}U_b + \frac{2}{15}U_\beta$$

$$- .7(1000)\left[\frac{25}{15}\left(\frac{6867}{12600}w_8 + \frac{4340}{12600}w_7 - \frac{25720}{12600}w_6 + \frac{15570}{12600}w_5 - \frac{1115}{12600}w_A + \frac{58}{12600}w_E\right)\right.$$

$$- \frac{50}{15}\left(\frac{6867}{12600}w_4 + \frac{4340}{12600}w_3 - \frac{25720}{12600}w_2 + \frac{15570}{12600}w_1 - \frac{1115}{12600}w_B + \frac{58}{12600}w_F\right)$$

$$+ \frac{23}{15}\left(\frac{6867}{12600}w_d + \frac{4340}{12600}w_c - \frac{25720}{12600}w_b + \frac{15570}{12600}w_a - \frac{1115}{12600}w_C + \frac{58}{12600}w_G\right)$$

$$\left. + \frac{2}{15}\left(\frac{6867}{12600}w_\delta + \frac{4340}{12600}w_\gamma - \frac{25720}{12600}w_\beta + \frac{15570}{12600}w_\alpha - \frac{1115}{12600}w_D + \frac{58}{12600}w_H\right)\right] = 0$$

$$450U_6 - 900U_2 + 414U_b + 36U_\beta$$

$$- 171675U_8 - 108500w_7 + 643000w_6 - 389250w_5 + 27875w_A - 1450w_E$$

$$+ 343350w_4 + 217000w_3 - 1286000w_2 + 778500w_1 - 55750w_B + 2900w_F$$

$$- 157941w_d - 99820w_c + 591560w_b - 358110w_a + 25645w_C - 1334w_G$$

$$- 13734w_\delta - 8680w_\gamma + 51440w_\beta - 31140_\alpha + 2230w_D - 116w_H = 0$$

Fourth of group [8]

$$\frac{25}{15}U_5 - \frac{50}{15}U_1 + \frac{23}{15}U_a + \frac{2}{15}U_\alpha$$

$$- .7(1000)\left[\frac{25}{15}\left(-\frac{83216}{25200}w_8 + \frac{141330}{25200}w_7 - \frac{56640}{25200}w_6 - \frac{23785}{25200}w_5 + \frac{23120}{25200}w_A - \frac{809}{25200}w_E\right)\right.$$

$$- \frac{50}{15}\left(\frac{83216}{25200}w_4 + \frac{141330}{25200}w_3 - \frac{56640}{25200}w_2 - \frac{23785}{25200}w_1 + \frac{23120}{25200}w_B - \frac{809}{25200}w_F\right)$$

$$+ \frac{23}{15}\left(-\frac{83216}{25200}w_d + \frac{141330}{25200}w_c - \frac{56640}{25200}w_b - \frac{23785}{25200}w_a + \frac{23120}{25200}w_C - \frac{809}{25200}w_G\right)$$

$$\left. + \frac{2}{15}\left(-\frac{83216}{25200}w_\delta + \frac{141330}{25200}w_\gamma - \frac{56640}{25200}w_\beta - \frac{23785}{25200}w_\alpha + \frac{23120}{25200}w_D - \frac{809}{25200}w_H\right)\right] = 0$$

$$900U_5 - 1800U_1 + 828U_a + 72U_\alpha$$

$$+ 2080400w_8 - 3533250w_7 + 1416000w_6 + 594625w_5 - 578000w_A + 20225w_E$$

$$- 4160800w_4 + 7066500w_3 - 2832000w_2 - 1189250w_1 + 1156000w_B - 40450w_F$$

$$+ 1913968w_d - 3250590w_c + 1302720w_b + 547055w_a - 531760w_C + 18607w_G$$

$$+ 166432w_\delta - 282660w_\gamma + 113280w_\beta + 47570w_\alpha - 46240w_D + 1618w_H = 0$$

Fifth of group [8]

$$\frac{25}{15}U_A - \frac{50}{15}U_B + \frac{23}{15}U_C + \frac{2}{15}U_D$$

$$\begin{aligned} - .7(1000)\Bigg[& \frac{25}{15}\left(\frac{250677}{12600}w_8 - \frac{436240}{12600}w_7 + \frac{284720}{12600}w_6 - \frac{121320}{12600}w_5 + \frac{15515}{12600}w_A + \frac{6648}{12600}w_E\right) \\ & - \frac{50}{15}\left(\frac{250677}{12600}w_4 - \frac{436240}{12600}w_3 + \frac{284720}{12600}w_2 - \frac{121320}{12600}w_1 + \frac{15515}{12600}w_B + \frac{6648}{12600}w_F\right) \\ & + \frac{23}{15}\left(\frac{250677}{12600}w_d - \frac{436240}{12600}w_c + \frac{284720}{12600}w_b - \frac{121320}{12600}w_a + \frac{15515}{12600}w_C + \frac{6648}{12600}w_G\right) \\ & + \frac{2}{15}\left(\frac{250677}{12600}w_\delta - \frac{436240}{12600}w_\gamma + \frac{284720}{12600}w_\beta - \frac{121320}{12600}w_\alpha + \frac{15515}{12600}w_D + \frac{6648}{12600}w_H\right)\Bigg] = 0 \end{aligned}$$

$$450U_A - 900U_B + 414U_C + 36U_D$$

$$- 6266925w_8 + 10906000w_7 - 7118000w_6 + 3033000w_5 - 387875w_A - 166200w_E$$

$$+ 12533850w_4 - 21812000w_3 + 14236000w_2 - 6066000w_1 + 775750w_B + 332400w_F$$

$$- 5765571w_d + 10033520w_c - 6548560w_b + 2790360w_a - 356845w_C - 152904w_G$$

$$- 501354w_\delta + 872480w_\gamma - 569440w_\beta + 242640w_\alpha - 31030w_D - 13296w_H = 0$$

Sixth of group [8]

$$\frac{3969}{630}U_d - \frac{7056}{630}U_c + \frac{5040}{630}U_b - \frac{3240}{630}U_a + \frac{1231}{630}U_C + \frac{56}{630}U_G$$

$$- .7(1000)\left[\frac{3969}{630}\left(\frac{235}{90}w_8 - \frac{270}{90}w_4 - \frac{27}{90}w_d + \frac{62}{90}w_\delta\right)\right.$$

$$- \frac{7056}{630}\left(\frac{235}{90}w_7 - \frac{270}{90}w_3 - \frac{27}{90}w_c + \frac{62}{90}w_\gamma\right)$$

$$+ \frac{5040}{630}\left(\frac{235}{90}w_6 - \frac{270}{90}w_2 - \frac{27}{90}w_b + \frac{62}{90}w_\beta\right)$$

$$- \frac{3240}{630}\left(\frac{235}{90}w_5 - \frac{270}{90}w_1 - \frac{27}{90}w_a + \frac{62}{90}w_\alpha\right)$$

$$+ \frac{1231}{630}\left(\frac{235}{90}w_A - \frac{270}{90}w_B - \frac{27}{90}w_C + \frac{62}{90}w_D\right)$$

$$\left.+ \frac{56}{630}\left(\frac{235}{90}w_E - \frac{270}{90}w_F - \frac{27}{90}w_G + \frac{62}{90}w_H\right)\right] = 0$$

$$3572.1U_d - 6350.4U_c + 4536U_b - 2916.U_a + 1107.9U_C + 50.4U_G$$

$$- 6529005w_8 + 11607120w_7 - 8290800w_6 + 5329800w_5 - 2024995w_A - 92120w_E$$

$$+ 7501410w_4 - 13335840w_3 + 9525600w_2 - 6123600w_1 + 2326590w_B + 105840w_F$$

$$+ 750141w_d - 1333584w_c + 952560w_b - 612360w_a + 232659w_C + 10584w_G$$

$$- 1722546w_\delta + 3062304w_\gamma - 2187360w_\beta + 1406160w_\alpha - 534254w_D - 24304w_H = 0$$

Seventh of group [8]

$$\frac{3969}{630}U_4 - \frac{7056}{630}U_3 + \frac{5040}{630}U_2 - \frac{3240}{630}U_1 + \frac{1231}{630}U_B + \frac{56}{630}U_F$$

$$\begin{aligned} - .7(1000)\Bigg[&\frac{3969}{630}\left(\frac{200}{180}w_8 - \frac{405}{180}w_4 + \frac{216}{180}w_d - \frac{11}{180}w_\delta\right) \\ &- \frac{7056}{630}\left(\frac{200}{180}w_7 - \frac{405}{180}w_3 + \frac{216}{180}w_c - \frac{11}{180}w_\gamma\right) \\ &+ \frac{5040}{630}\left(\frac{200}{180}w_6 - \frac{405}{180}w_2 + \frac{216}{180}w_b - \frac{11}{180}w_\beta\right) \\ &- \frac{3240}{630}\left(\frac{200}{180}w_5 - \frac{405}{180}w_1 + \frac{216}{180}w_a - \frac{11}{180}w_\alpha\right) \\ &+ \frac{1231}{630}\left(\frac{200}{180}w_A - \frac{405}{180}w_B + \frac{216}{180}w_C - \frac{11}{180}w_D\right) \\ &+ \frac{56}{630}\left(\frac{200}{180}w_E - \frac{405}{180}w_F + \frac{216}{180}w_G - \frac{11}{180}w_H\right)\Bigg] = 0 \end{aligned}$$

$$\begin{aligned} &7144.2U_4 - 12700.8U_3 + 9072.U_2 - 5832.U_1 + 2215.8U_B + 100.8U_F \\ &- 5556600w_8 + 9878400w_7 - 7056000w_6 + 4536000w_5 - 1723400w_A - 78400w_E \\ &+ 11252115w_4 - 20003760w_3 + 14288400w_2 - 9185400w_1 + 3489885w_B + 158760w_F \\ &- 6001128w_d + 10668672w_c - 7620480w_b + 4898880w_a - 1861272w_C - 84672w_G \\ &+ 305613w_\delta - 543312w_\gamma + 388080w_\beta - 249480w_\alpha + 94787w_D + 4312w_H = 0 \end{aligned}$$

The values of the unknowns and the check as computed by the digital computer, and the right hand side of the equations for reference purposes, are shown on the following pages. The equations are in the order presented on the previous pages.

UNKNOWNS	VALUES OF UNKNOWNS	CHECK	RHS OF EQUATIONS
U_1	2-37421546	-41495424	0
U_2	2-46150031	-1-23675308	0
U_3	2-56521068	-93942830	0
U_4	2-55833804	22138032	0
U_5	2-25630691	1-15993863	0
U_6	2-73512452	-21289384	0
U_7	3-13551100	-69024450	0
U_8	3-16853459	-1-97370539	0
U_a	2-42389301	1-26723355	0
U_b	2-15392765	1 17006855	0
U_c	2 14531152	-2-40637173	0
U_d	2 95498595	-3-12333255	0
U_A	2-20545537	-3 28961536	0
U_B	2-21982002	-2-21057129	0
U_D	3 75525598	-2-93404614	0
U_F	3-22164921	-2 12663097	0
U_G	4-37711825	-2 53857883	0
U_α	2-28841752	-2-18310547	0
U_β	2 43765879	-2 62136229	0
U_γ	3 14222960	-3 49925904	0
U_δ	3 77044013	-2 29305992	0
w_1	-1 11740179	-3 92653071	0
w_2	-1 33691164	-2 10459937	0
w_3	-1 61528525	-3 82015991	0
w_4	-1 77139796	-2-13986714	0

UNKNOWNS	VALUES OF UNKNOWNS	CHECK	RHS OF EQUATIONS
w_a	-1 36770965	-2-28673909	0
w_b	10516378	-3 76436695	0
w_c	19030313	-3 64926260	0
w_d	24682922	-2-12385740	0
w_B	-1 15002952	-6-39836254	0
w_C	-1 58933967	-3 31160813	0
w_D	-1 91000149	-5 22642754	0
w_E	-1-16688792	-6 73625176	0
w_F	-15277891	5 67499939	67500
w_G	-79002871	-1 68359375	0
w_H	-1-50027561	-3-18310547	0
w_α	-1 40768464	-20703125	0
w_β	18294061	56054688	0
w_γ	35986091	85156250	0
w_δ	50283278	-1 56640625	0

For example 1-15993863 equals -1.5993863

-21289384	"	- .21289384
-1-97370539	"	- .097370539
-3 28961536	"	.00028961536

Table 1. Values of Unknowns U and w as Presented in Holl's Article (1). Poisson's Ratio Equals Three Tenths.

Net Point	U	w	Mx	My
8	-.49672P		-.49672P	-.01498P
7	-.37352P		-.37352P	-.03780P
6	-.22672P		-.22672P	-.06802P
5	-.12600P		-.12600P	-.11206P
A	-.04992P		-.04992P	-.14902P
4	-.13091P	$.06209Pa^2/N$	-.21714P	.04696P
3	-.15847P	$.04669Pa^2/N$	-.16670P	-.03931P
2	-.13089P	$.02834Pa^2/N$	-.11477P	-.05539P
1	-.08634P	$.01575Pa^2/N$	-.07780P	.03444P
B	-.04948P	$.00624Pa^2/N$	-.06432P	
d	.29002P	$.18773Pa^2/N$	0	.37703P
c	.00143P	$.13594Pa^2/N$	0	.00186P
b	-.05205P	$.08364Pa^2/N$	0	-.06766P
a	-.03900P	$.04993Pa^2/N$	0	-.05070P
C	0	$.03015Pa^2/N$	0	0
D		$.05406Pa^2/N$		
G		$.01037Pa^2/N$		
H		$.07275Pa^2/N$		
F		$-.00857Pa^2/N$		
α		$.07993Pa^2/N$		
β		$.13336Pa^2/N$		
γ		$.22534Pa^2/N$		
δ		$.34445Pa^2/N$		

Table 2. Calculation of Bending Moments Mx and My Using Values of Deflection, w, from Holl's Article (1). Poisson's Ratio Equals Three Tenths.

Net Point	① $\partial^2 w/\partial x^2$	② $\partial^2 w/\partial y^2$
8	2(.06209) = .12418	0
7	2(.04669) = .09338	0
6	2(.02834) = .05668	0
5	2(.01575) = .03150	0
A	2(.00624) = .01248	0†
4	.18773-2(.06209)=.06355	2(.04669)-2(.06209)=-.03080
3	.13594-2(.04669)=.04256	.02934+.06209-2(.04669)=-.00295
2	.08364-2(.02834)=.02696	.01575+.04669-2(.02834)=.00576
1	.04993-2(.01575)=.01843	.00624+.02834-2(.01575)=.00308
B	.03015-2(.00624)=.01767	-.00857+.01575-2(.00624)=-.00530
d	.34445+.06209-2(.18773)=.03108	2(.13594)-2(.18773) =-.10358
c	.22534+.04669-2(.13594)=.00015	.08364+.18773-2(.13594)=-.00051
b	.13336+.02834-2(.08364)=-.00558	.04993+.13594-2(.08364)=.01859
a	.07993+.01575-2(.04993)=-.00418	.03015+.08364-2(.04993)=.01393

† w_A and w_5 equal zero and Holl has assumed w_E equal to zero both here and in the last equation of group [8].

Table 2, Cont'd. Calculation of Bending Moments Mx and My Using Values of Deflection, w, from Holl's Article (1). Poisson's Ratio Equals Three Tenths.

Net Point	3 v 1 = .3 1	4 v 2 = .3 2	5 1 + 4	6 2 + 3	7 -4 5 = Mx	8 -4 6 = My
8	.037254	0	.12418	.037254	-.49672	-.149016
7	.028014	0	.09338	.028014	-.37352	-.112056
6	.017004	0	.05668	.017004	-.22672	-.068016
5	.009450	0	.03150	.009450	-.12600	-.0378
A	.003744	0	.01248	.003744	-.04992	-.014976
4	.019065	-.00924	.05431	-.011735	-.21724	.04694
3	.012768	-.000885	.041675	.009818	-.16670	-.039272
2	.008088	.001728	.028688	.013848	-.114752	-.055392
1	.005529	.000924	.019354	.008609	-.077416	-.034436
B	.005301	-.00159	.016080	.000001	-.064320	-.000004
d	.009324	-.031074	.000006	-.094256	-.000024	.377024
c	.000045	-.000153	-.000003	-.000465	.000012	.00186
b	-.001674	.005577	-.000003	.016916	.000012	-.067664
a	-.001254	.004179	-.000001	.012676	.000004	-.050704

Table 3. Comparison of Coefficients of "U" and "w"

				Coefficients of "U", U = coefficient x P							
Solution	1	2	3	4	5	6	7	8	9	b	
Holl	-.08634	-.13089	-.15847	-.13091	-.12600	-.22672	-.37352	-.49672	-.039	-.05205	
1	-.04998	-.10477	-.14085	-.11630	-.05695	-.19121	-.35524	-.48361	-.02366	-.03702	
2	-.05698	-.10975	-.14422	-.11910	-.06985	-.19795	-.35875	-.48616	-.02657	-.03985	
3	-.14969	-.18460	-.22608	-.22334	-.10252	-.29405	-.54204	-.67414	-.16956	-.06157	
				Coefficients of "U", U = coefficient x P							
Solution	c	d	A	B	D	F	G	α	β	γ	δ
Holl	.00143	.29002	-.04992	-.04948							
1	.01292	.30011	.06577	-.01456	.02093	-.07403	.01729	-.00765	-.03257	-.07055	1.29090
2	.01072	.29819	.03118	-.02176	.02513	-.06123	.02320	-.00945	-.03379	-.07122	1.29041
3	.05812	.38199	-.08218	-.08793	3.02102	-.88660	-15.08473	-.11537	.17506	.56892	3.08176
				Coefficients of "w", w = coefficient x Pa^2/N							
Solution	1	2	3	4	a	b	c	d	B	C	
Holl	.01575	.02834	.04669	.06209	.04993	.08364	.13594	.18773	.00624	.03015	
1	.00711	.02390	.04441	.06045	.02529	.07027	.12848	.18207	-.00822	-.01124	
2	.00873	.02474	.04484	.06077	.02999	.07284	.12992	.18317	-.00557	-.00337	
3	.01174	.03369	.06153	.07714	.03677	.10516	.19030	.24683	.01500	.05893	
				Coefficients of "U", w = coefficient x Pa^2/N							
Solution	D	F	G	H	α	β	γ	δ	E		
Holl	.05406	-.00857	.01037	.07275	.07993	.13336	.22534	.34445	0		
1	-.01426	-.02512	-.04778	-.05834	-.04093	.11268	.21394	.33584	0		
2	-.00116	-.02220	-.03672	-.03632	.04841	.11667	.21614	.33751	.00334		
3	.09100	-.15278	-.79003	-.05003	.04077	.18294	.35986	.50283	-.01669		

Table 4. Comparison of Values of Maximum Deflection and Maximum Moment of a Simply Supported, Uniformly Loaded, Square Plate

Source	$w_{max} \frac{Eh^3}{qa^4}$	Mx_{max}/qa^2
Exact solution on page 11 and from Timoshenko (14)	.044360	.0479
Finite difference solution with $\Delta x = \Delta y = a/4$ from Timoshenko (10).	.0440	.0457
Finite difference solution with $\Delta x = \Delta y = a/6$ from page 41.	.044208	.046875
Polynomial solution with $\Delta x = \Delta y = a/6$ from page 43.	.044375	.047885

Table 5. Sixth Order Polynomial Coefficients (Ref. pg.39)

Coefficient / Derivative	y_o	y_1	y_2	y_3	x
	0	0	0	0	0
$\frac{dy}{dx}$	-80/60	65/60	16/60	-1/60	λ
	25/15	-50/15	23/15	2/15	2λ
	-400/60	675/60	-432/60	157/60	3λ
	-245/90	270/90	-27/90	2/90	0
$\frac{d^2y}{dx^2}$	200/180	-405/180	216/180	-11/180	λ
	235/90	-270/90	-27/90	62/90	2λ
	-5080/180	8235/180	-4104/180	949/180	3λ

Table 6. Tenth Order Polynomial Coefficients

(Ref. pgs. 48 and 49)

Coefficient / Derivative	y_o	y_1	y_2	y_3	y_4	y_5	x
$\frac{dy}{dx}$	0	0	0	0	0	0	0
	$-\frac{3024}{2520}$	$\frac{2184}{2520}$	$\frac{960}{2520}$	$-\frac{135}{2520}$	$\frac{16}{2520}$	$-\frac{1}{2520}$	λ
	$\frac{1323}{1260}$	$-\frac{2940}{1260}$	$\frac{1272}{1260}$	$\frac{378}{1260}$	$-\frac{35}{1260}$	$\frac{2}{1260}$	2λ
	$-\frac{1568}{840}$	$\frac{2940}{840}$	$-\frac{2688}{840}$	$\frac{1163}{840}$	$\frac{160}{840}$	$-\frac{7}{840}$	3λ
	$\frac{3969}{630}$	$-\frac{7056}{630}$	$\frac{5040}{630}$	$-\frac{3240}{630}$	$\frac{1231}{630}$	$\frac{56}{630}$	4λ
	$-\frac{127008}{2520}$	$\frac{220500}{2520}$	$-\frac{144000}{2520}$	$\frac{70875}{2520}$	$-\frac{28000}{2520}$	$\frac{7633}{2520}$	5λ
$\frac{d^2y}{dx^2}$	$-\frac{36883}{12600}$	$\frac{42000}{12600}$	$-\frac{6000}{12600}$	$\frac{1000}{12600}$	$-\frac{125}{12600}$	$\frac{8}{12600}$	0
	$\frac{38304}{25200}$	$-\frac{73430}{25200}$	$\frac{39040}{25200}$	$-\frac{4365}{25200}$	$\frac{480}{25200}$	$-\frac{29}{25200}$	λ
	$\frac{6867}{12600}$	$\frac{4340}{12600}$	$-\frac{25720}{12600}$	$\frac{15570}{12600}$	$-\frac{1115}{12600}$	$\frac{58}{12600}$	2λ
	$-\frac{83216}{25200}$	$\frac{141330}{25200}$	$-\frac{56640}{25200}$	$-\frac{23785}{25200}$	$\frac{23120}{25200}$	$-\frac{809}{25200}$	3λ
	$\frac{250677}{12600}$	$-\frac{436240}{12600}$	$\frac{284720}{12600}$	$-\frac{121320}{12600}$	$\frac{15515}{12600}$	$\frac{6648}{12600}$	4λ
	$-\frac{6932016}{25200}$	$\frac{11961250}{25200}$	$-\frac{7640000}{25200}$	$\frac{3549375}{25200}$	$-\frac{1130000}{25200}$	$\frac{191391}{25200}$	5λ

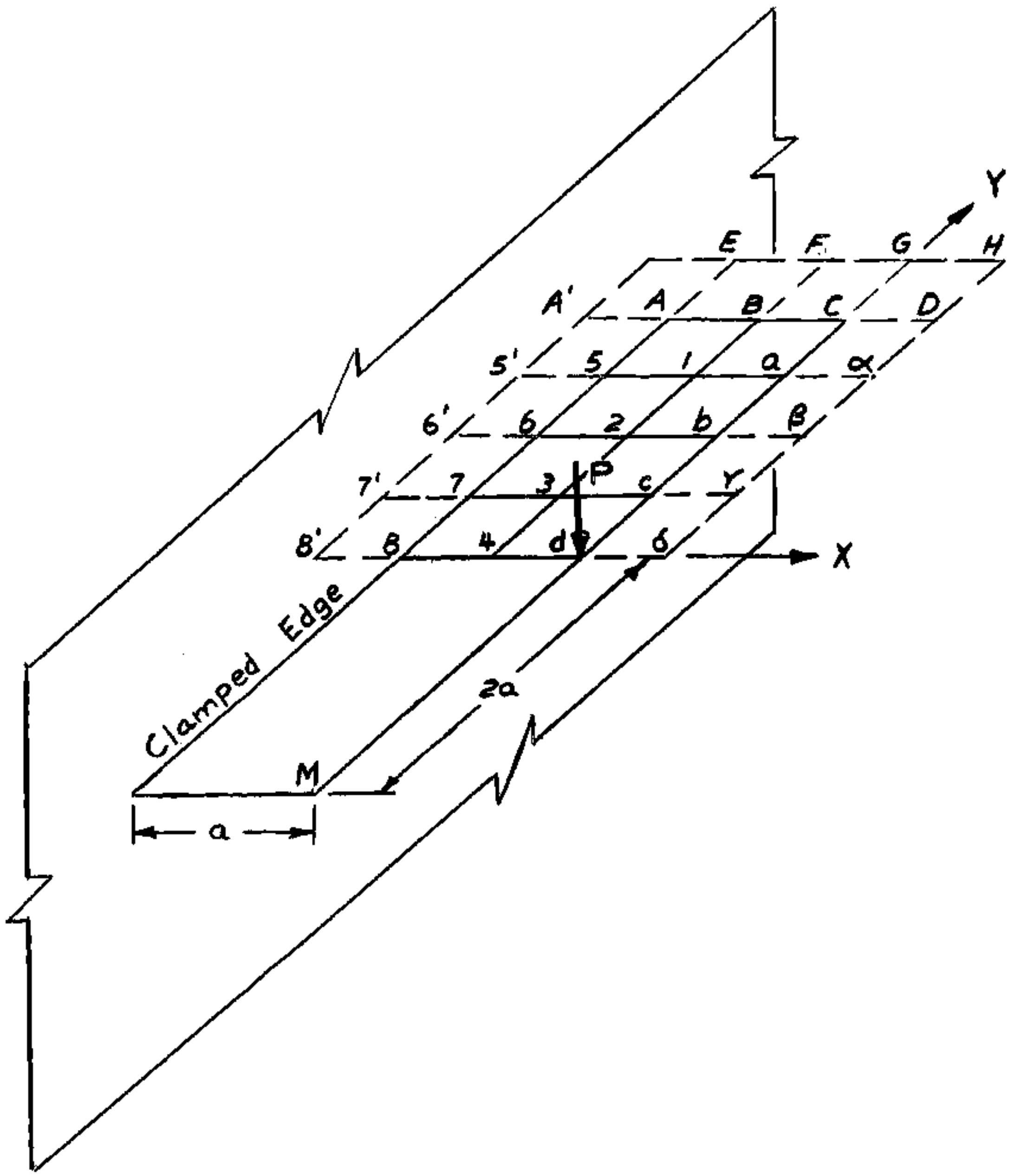

Figure 1. Cantilever Plate with Concentrated Edge Load Showing Axes and Net Points as Shown in Holl's Article (1).

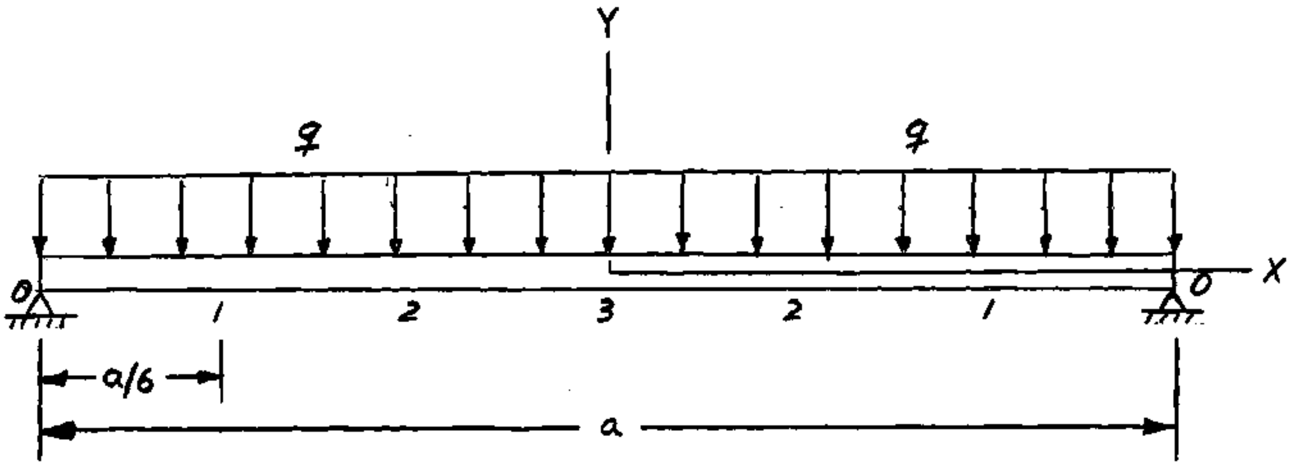

Figure 2. Uniformly Loaded, Long Rectangular Plate Showing Axes and Net Points

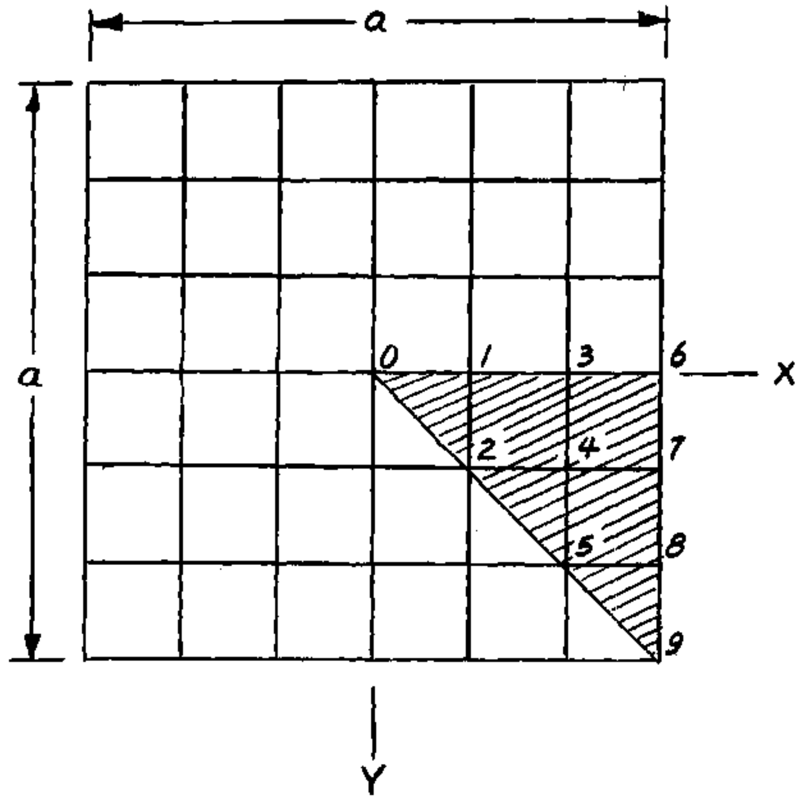

Figure 3. Uniformly Loaded, Simply Supported Square Plate Showing Axes and Net Points. Only One Eigth of Plate Shown Due to Symmetry

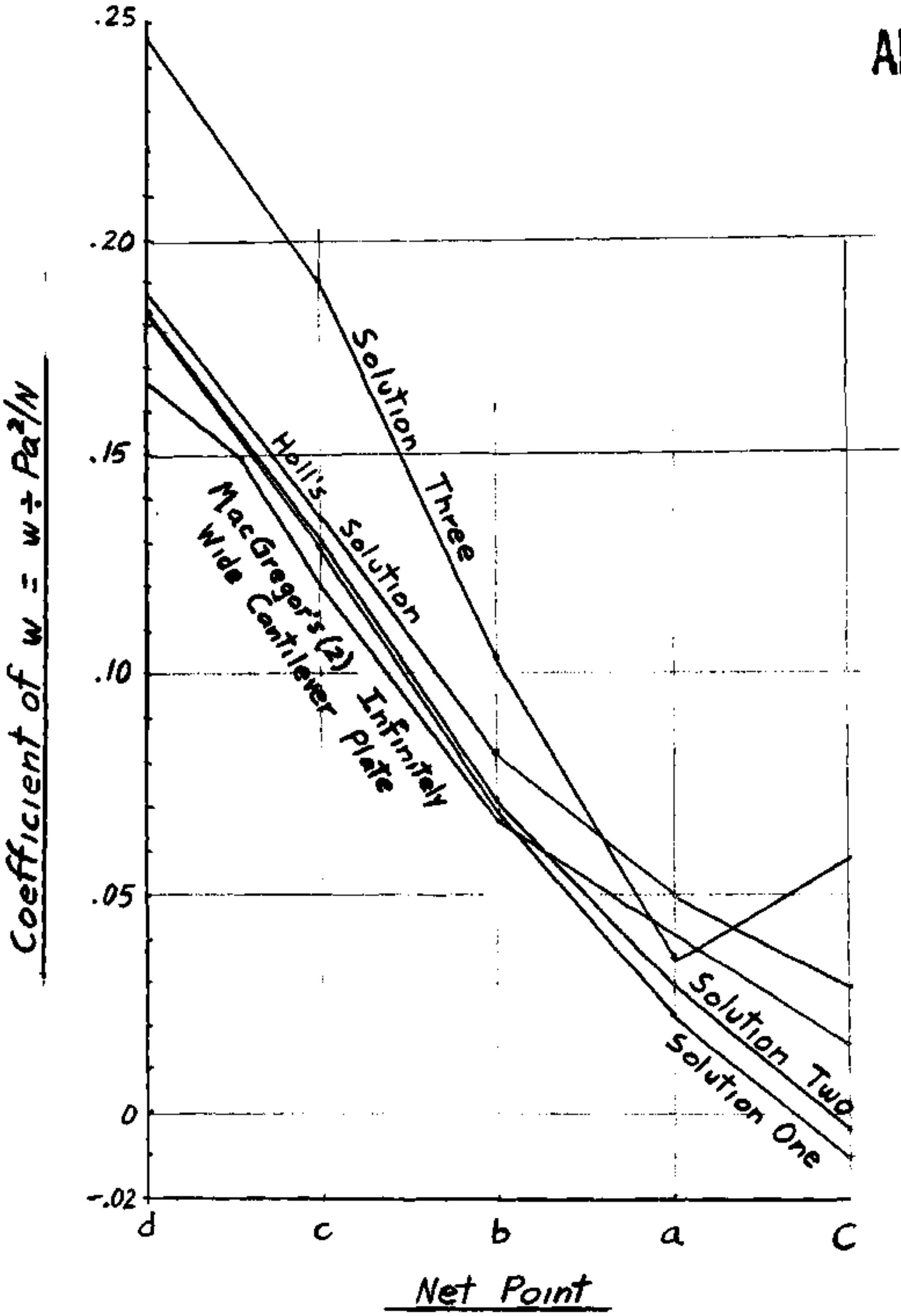

Figure 4. Comparison of Edge Deflections of Cantilever Plate with Concentrated Edge Load